QBUS2310 Management Science

THE UNIVERSITY OF SYDNEY · LEVEL 2 · SEMESTER 2 2026

Full guide + ask Sia about QBUS2310

SIDE 1/2 FINAL PRIORITIES · DEFINITIONS · FORMULAS · DECISION RULES · CERTIFICATES — **EXAM REVISION SHEET** — 40% final written exam · two hours

01 · Exam map

COURSE.JSON

Final 40 | A0 8 | MST 20 | A1 12 | A2 20

ASSESSMENT	WEIGHT
Final exam Written exam	40%
Assignment 0 A written assignment submit	8%
Mid-Semester Test A mid-semester test	20%
Assignment 1 A written assignment submit	12%
Assignment 2 A written assignment submit	20%

Permitted materials: not confirmed for this offering in the course materials we hold — check the exam instructions on the LMS. *[BRIEF]*

02 · The answer spine

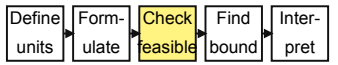
CH 01 · 09

SIX MOVES, IN ORDER

`Define choices + units → separate parameters → state objective → write limits + domains → test feasibility → certify + interpret.`

★ ***Decision variable*** = controllable quantity. ***Parameter*** = fixed information. An ***objective*** ranks plans; a ***constraint*** decides which plans exist.

CLAIM	MINIMUM CHECK
Feasible	Every row and domain passes substitution
LP optimum	Feasible candidate + valid bound
Network	Conservation at every node
Game	Probabilities sum to 1; all responses checked
Integer	Incumbent compared with relaxation bound



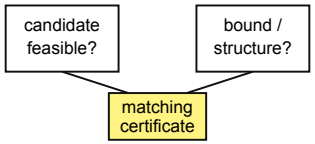
verify before concluding

Guide figure redraw: the auditable decision chain. *[front fig. 2]*

Proof obligation

COURSE OVERVIEW

LP	feasible point + bound
GAME	guarantee against every response
INTEGER	incumbent + relaxation bound
NETWORK	all balances + capacity/cut
RISK	loss + probability convention



Guide verification theme: a candidate and an independent check must meet.

03 · Linear model test

CH 01

GENERIC LP

`max/min cᵀx  subject to  Ax ≤, =, or ≥ b; state x ≥ 0, x free, or other domains explicitly.`

AUDIT	QUESTION	FAILURE SIGNAL
Units	Does each row use one unit?	Unlike quantities added
Proportionality	Double activity → double effect?	Discount/setup/learning
Additivity	Effects sum without interaction?	Cross-product/interference
Domain	Continuous, integer, binary, free?	Missing sign/restriction
Direction	Capacity ≤? requirement ≥?	Operational sentence reversed

Infeasible: no point satisfies all restrictions; identify conflicting rows. ***Unbounded***: an improving feasible direction continues without limit; inspect omitted capacity or sign restrictions. These diagnoses are different.

04 · Algebra + convexity

CH 02

DIMENSION RULE

`(m×n)(n×p) → m×p. Inner dimensions must agree; AB generally differs from BA.`

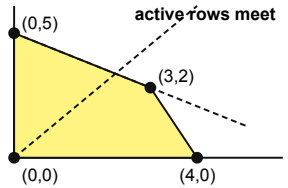
CONVEX COMBINATION

`x(θ)=θx¹+(1−θ)x²,  0≤θ≤1. If x¹,x² are feasible for linear inequalities, every x(θ) is feasible.`

05 · Geometry + standard form

CH 03

ACTIVE ROW	Equality holds at the candidate.
SLACK, ≤ ROW	$b-a^{\mathsf{T}}x$: unused amount.
SURPLUS, ≥ ROW	$a^{\mathsf{T}}x-b$: excess above requirement.
BASIC SOLUTION	Enough independent equalities determine a point; feasibility is separate.
DEGENERATE BFS	More active restrictions than needed to determine it.



Guide figure redraw: feasible vertices for the source LP. *[front fig. 7; ch. 3]*

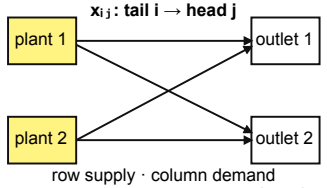
06 · Model-family chooser

CH 04

STORY CUE	VARIABLES + CORE ROW
Production/resource	activity quantities; total use ≤ availability
Diet/service	chosen quantities; delivered amount ≥ need
Transport	x_{ij} shipped; one source and destination balance
Workforce coverage	staff starting/working; coverage ≥ demand
Inventory	production, stock; opening + production − demand = closing
Cash flow	borrowing/investment; preserve period timing

Build by entity and period, not by copying a template: one row per source, destination, day, period or month. Every variable needs an entity, time index where relevant, and physical or monetary unit. A repayment or carry belongs to its actual later period.

Hard constraint: legal/physical rule that cannot be bought away. ***Soft target***: deviation may be penalised. Putting a hard limit only in the objective can admit an operationally illegal plan.



Guide figure redraw: transportation balances. *[front fig. 8]*

07 · Balance templates

CH 04

TRANSPORT

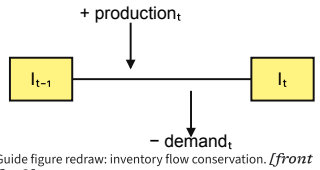
$\sum_j x_{ij} = \text{supply}_i$; $\sum_i x_{ij} = \text{demand}_j$; $x_{ij} \geq 0$; minimise $\sum_i \sum_j c_{ij} x_{ij}$.

INVENTORY

$I_{t-1} + \text{production}_t - \text{demand}_t = I_t$. Keep opening and closing stock on the correct side and period.

COVERAGE

$\sum$ shifts covering period t $x_s \geq \text{demand}_t$. Cyclic schedules must wrap coverage across the boundary.



Guide figure redraw: inventory flow conservation. *[front fig. 9]*

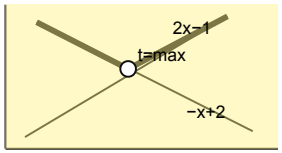
08 · Linearisation

CH 05

EXPRESSION / GOAL	EXACT LP PATTERN
min $\max_k f_k(x)$	min t; $t \geq f_k(x)$ for every k
min $\lvert g(x) \rvert$	min t; $t \geq g(x)$, $t \geq -g(x)$
max $\min_k f_k(x)$	max t; $t \leq f_k(x)$ for every k
convex piecewise cost	upper-envelope rows; objective pushes down

State what the auxiliary variable bounds and why the objective makes it tight. Equivalence must work both ways: each original feasible plan extends to the reformulation, and each reformulated solution recovers the intended original value.

Big-M: derive a finite linking bound from model limits. Too small cuts off feasible plans; unnecessarily large M weakens the relaxation. A concave cost treated with the same convex-envelope construction may be only a relaxation and can require binary segment selection.



minimise t: squeeze to upper envelope

Guide figure redraw: epigraph reformulation. *[front fig. 10]*

09 · Duality certificate

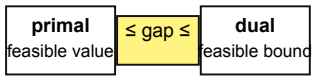
CH 06

CANONICAL PAIR USED IN THE GUIDE EXAMPLE

`Primal max: cᵀx, Ax≤b, x≥0.`
`Dual min: bᵀy, Aᵀy≥c, y≥0.`

Weak duality: every feasible primal value is no larger than every feasible dual value for this max/min pair. Equal feasible values certify both optima. The duality gap is bound minus incumbent.

COMPLEMENTARY-SLACKNESS CHECK	AT OPTIMUM
Primal row has positive slack	Associated dual multiplier = 0
Dual row has positive slack	Associated primal variable = 0
Multiplier > 0	Associated primal row binds
Primal variable > 0	Associated dual row binds



equality ⇒ optimum

Guide figure redraw: weak-duality bound. *[front fig. 12]*

10 · Zero-sum games

CH 07

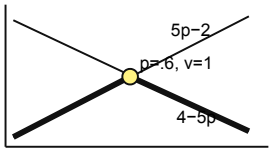
ROW PLAYER'S MAXIMIN LP

`max v;  expected payoff against each column ≥ v;  ∑ᵢpᵢ=1; pᵢ≥0.`

COLUMN PLAYER'S MATCHING BOUND

`min w;  expected payoff for each row ≤ w;  ∑ⱼqⱼ=1; qⱼ≥0.`

A ***mixed strategy*** is a probability distribution over pure actions. In a small 2×2 game, write one expected-payoff line per opposing pure action; the row player maximises the lower envelope. Equalise active responses, but still check endpoints and 0≤p≤1. Matching row guarantee and column upper bound certify the game value.



maximise the worst response

Guide figure redraw: mixed-strategy lower envelope. *[front figs. 14–15]*

11 · Integer models

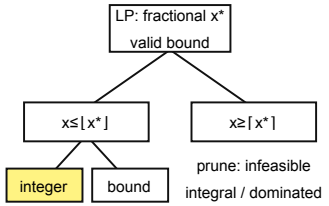
CH 08

CUE	MODEL / RULE
Indivisible count	$x \in \mathbb{Z}_+$
Yes/no selection	$y \in \{0,1\}$
Activate quantity	$0 \leq x \leq My$, with justified M
Cover each requirement	sum of selections ≥1
Partition exactly once	sum of selections =1

The LP relaxation removes integrality but retains the linear rows. Its optimum is a ceiling for maximisation and a floor for minimisation. It need not be implementable. The integrality gap separates the relaxation bound from the best integer value and depends on the formulation.

BRANCH-AND-BOUND

1. Solve the node relaxation.
2. If infeasible, prune. If integral, update the incumbent.
3. If its bound cannot beat the incumbent, prune by bound.



Guide figure redraw: branch-and-bound logic. *[front fig. 18]*

12 · Network models

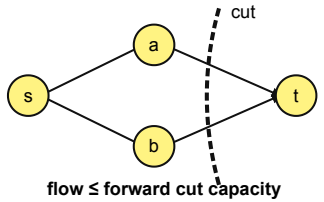
CH 09

DIRECTED ARC MODEL

`xᵢⱼ≥0;  0≤xᵢⱼ≤uᵢⱼ;  minimise ∑cᵢⱼxᵢⱼ. At node i: inflow − outflow = stated net demand/supply convention.`

FAMILY	STRUCTURAL CHECK
Minimum-cost flow	Balances + capacity; minimise arc cost
Shortest path	One unit sent source→sink; path cost
Assignment	One-to-one row and column equations
Maximum flow	Maximise source–sink flow
Minimum cut	Forward cut capacity upper-bounds any flow

Fix direction before equations: the same x_{ij} leaves tail i and enters head j. At a transshipment node, inflow=outflow. Network integrality conclusions require the relevant structure and integer data; do not generalise them to arbitrary LPs.



flow ≤ forward cut capacity

Guide figure redraw: conservation and cut bound. *[front figs. 19–20]*

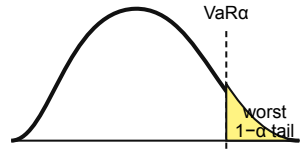
13 · Uncertainty + risk

CH 10

CRITERION	WHAT IT PROTECTS	BOUNDARY
Expected value	Probability-weighted average	Not rare-loss severity
Worst case	Most adverse admitted outcome	Only inside stated set
Recourse	Action after scenario observed	First-stage choice cannot vary by scenario
VaRα	Loss quantile threshold	State confidence + convention
CVaRα	Mean loss in worst 1−α tail	Handle mass at VaR

SCENARIO EXPECTATION

$E[L]=\sum_s p_s L_s$, with $p_s \geq 0$ and $\sum_s p_s = 1$.



CVaRα = mean loss in shaded tail

Guide figure redraw: VaR threshold and CVaR tail. *[front fig. 23]*

QBUS2310 Management Science · Self-test drills

WORKED ANSWER PATHS · JUDGEMENT CARDS · TRAPS · EXAM-MORNING PLAN

Full guide + ask Sia about QBUS2310

SIDE 2/2 FIVE SOURCE WORKED PATHS · IF X → DO Y · CLASSIC TRAPS · FAST RECALL

SELF-TEST BEFORE READING ANSWER

Every number below is from the guide

14 · Drill: LP + dual CH 03 · 06

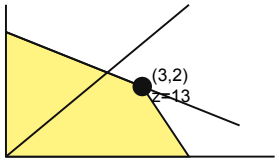
Prompt. Maximise 3x+2y subject to x+y≤5, 2x+y≤8, x,y≥0. Find and verify the optimum.

ANSWER PATH

1. Intersect boundaries and axes; reject any infeasible intersection.
2. Vertices: (0,0), (4,0), (3,2), (0,5).
3. Objective values: 0, 12, 13, 10. Incumbent: (3,2), value 13.
4. Dual: min 5u+8v subject to u+2v≥3, u+v≥2, u,v≥0.
5. Take u=v=1: dual feasible, value 13. Equal feasible values certify optimality.

(X,Y)	X+Y≤5	2X+Y≤8	3X+2Y
(0,0)	0 ✓	0 ✓	0
(4,0)	4 ✓	8 ✓	12
(3,2)	5 ✓	8 ✓	13
(0,5)	5 ✓	5 ✓	10

SAY THE CERTIFICATE *"Primal feasible value 13 equals dual feasible bound 13; hence both are optimal." Vertex enumeration alone works here because the feasible polygon is bounded, but the dual equality is the shorter independent proof.*

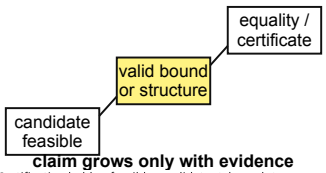


both resource rows active

Worked geometry: incumbent at the binding-row intersection. *[ch. 3 example]*

27 · One-minute proof cards CHS 02 · 06 · 08 · 09

CLAIM	RECITABLE PROOF SKELETON
LP feasible set convex	Take feasible x^1, x^2 and $0\le\theta\le1$. Linearity preserves every inequality at $\theta x^1+(1-\theta)x^2$.
LP candidate optimal	Show primal feasibility; show dual feasibility; show equal values.
Node flow valid	Check arc bounds and every node balance.
Flow impossible	Exhibit a source–sink cut below required flow.



claim grows only with evidence

Certification ladder: feasible candidate → bound → conclusion. *[guide verification theme]*

16 · Drill: transportation CH 04

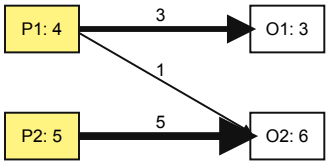
Prompt. Plants supply 4 and 5; outlets demand 3 and 6. Arc costs are 1, 4, 3, 2. Minimise shipping cost.

ANSWER PATH

1. Let x_{ij} be units shipped from plant i to outlet j; write two row and two column balances.
2. Let $a=x_{11}$. Balances force the four flows to be a, 4−a, 3−a, 2+a.
3. Nonnegativity gives 0≤a≤3.
4. Cost = a+4(4−a)+3(3−a)+2(2+a)=29−4a.
5. Cost falls as a rises: a=3; matrix [[3,1],[0,5]]; cost 17.

	OUTLET 1 DEMAND 3	OUTLET 2 DEMAND 6	SUPPLY
PLANT 1	3 @ 1	1 @ 4	4
PLANT 2	0 @ 3	5 @ 2	5
DEMAND	3	6	cost 17

AUDIT *All flows are nonnegative; both supplies and both demands balance; direct cost = 3+4+0+10=17.*



flows [[3,1],[0,5]]

Worked transport plan: thickness tracks positive flow. *[ch. 4 example]*

17 · If the story says... CH 04 · 09

"at most", capacity, budget	→ total use ≤ availability
"at least", service, nutrition	→ delivered amount ≥ requirement
stock carries forward	→ indexed inventory balance
directed routes	→ arc variables; tail/head node balances
one worker/item matched once	→ assignment row/column equations
legal/physical limit	→ hard row, not a mere objective penalty

Read every finished constraint aloud in operational units. A coefficient must answer "how much of this row's resource does one unit of this column's activity use?"

28 · Exam-morning plan STUDY STRATEGY

1. **Recall:** reproduce the answer spine, dual and envelope directions, prune rules and VaR/CVaR distinction.
2. **Formulate:** write variables with units, objective, rows and domains for one fresh story.
3. **Audit:** locate a planted units, index, timing or inequality defect by reading aloud.

18 · Drill: epigraph CH 05

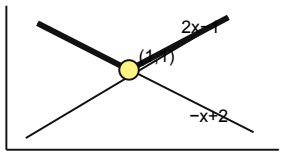
Prompt. Minimise max{2x−1, −x+2} for 0≤x≤3.

EXACT REFORMULATION

```
min t
t≥2x-1
t≥-x+2
0≤x≤3
```

1. t is an upper bound on both affine pieces.
2. Minimising t pushes it onto their upper envelope.
3. At the minimum, active pieces meet: 2x−1=−x+2.
4. 3x=3, so x=1 and t=1.
5. Check endpoints: their envelope values are larger. Answer: (x,t)=(1,1).

DIRECTION TRAP *Reversing either epigraph inequality no longer models the maximum. Inequalities alone are insufficient unless the objective pushes t towards the intended tight bound.*



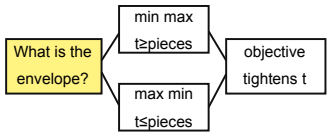
minimum of the upper envelope

Worked epigraph: active pieces equalise at x=1. *[ch. 5 example]*

19 · Linearisation traps CH 05 · 08

IF YOU SEE	DO	DO NOT
|g(x)| in a minimisation	t≥g and t≥−g	Use only one sign
min of max	Upper-bound all pieces	Reverse rows
max of min	Lower-bound by all pieces	Reuse epigraph direction
logical switch	Derive tight finite M	Choose a huge round M
fractional integer relaxation	Branch/cut/bound	Round and declare optimal

At each breakpoint, evaluate neighbouring branches and confirm the intended envelope selects the right value. Say whether the formulation is *exact* or an *approximation/relaxation*.



Judgement card: choose inequality direction from the envelope and objective. *[ch. 5]*

20 · Drill: zero-sum game CH 07

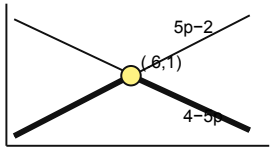
Prompt. Row payoff matrix [[3,−1],[−2,4]]. Find optimal mixes and value.

	C1	C2
R1	3	−1
R2	−2	4

1. Let p=P(R1). Against C1: 3p−2(1−p)=5p−2.
2. Against C2: −p+4(1−p)=4−5p.
3. Maximise the lower line; equalise: 5p−2=4−5p.
4. p=0.6; value v=1. Check endpoints p=0 and p=1.
5. For the column player, q=P(C1)=0.5 equalises both row payoffs at 1.

ANSWER

```
Row mix (0.6,0.4); column mix
(0.5,0.5); value 1. Both guarantees
meet, so the strategies are optimal.
```



lower envelope peaks where lines meet

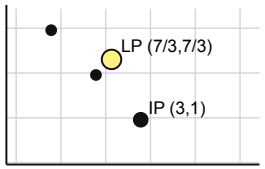
Worked game: the protected payoff is the lower line. *[ch. 7 example]*

21 · Drill: integer optimum CH 08

Prompt. Maximise 4x+3y subject to 2x+y≤7, x+2y≤7, x,y∈ℤ₊.

1. Relax integrality. Binding equations give x=y=7/3; relaxed value 49/3.
2. For this maximisation, 49/3 is an upper bound, not an implementable plan.
3. Nearby feasible lattice points: (3,1)→15; (1,3)→13; (2,2)→14.
4. Rounding up violates both rows; rounding down gives 14.
5. Integer optimum: (3,1), value 15. Gap = 49/3−15=4/3.

BOUND LANGUAGE *"Upper bound 49/3; incumbent 15; gap 4/3." Never call the relaxed point the integer solution.*



fractional bound ≠ feasible integer plan

Guide figure redraw: integrality gap and feasible lattice points. *[front fig. 17; ch. 8]*

22 · Drill: VaR + CVaR CH 10

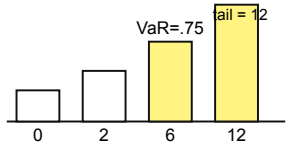
Prompt. Equal-probability losses are 0, 2, 6, 12. Compute $VaR_{0.75}$ and $CVaR_{0.75}$.

SORTED LOSS	0	2	6	12
CUMULATIVE P	.25	.50	.75	1.00

1. The smallest loss whose cumulative probability reaches .75 is 6.
2. Therefore $VaR_{0.75}=6$.
3. The worst tail has probability .25 and contains loss 12 only.
4. Therefore $CVaR_{0.75}=12$.
5. State the discrete convention: averaging every observation at or above VaR can wrongly include threshold mass.

INTERPRET

```
VaR is the threshold; CVaR is the mean
severity in the specified upper tail.
Here: 6 versus 12.
```



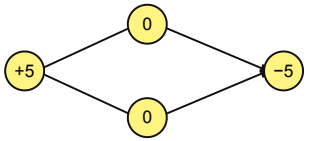
each outcome has probability .25

Worked risk distribution: threshold at 6, worst tail at 12. *[ch. 10 example]*

23 · Drill: network proof CH 09

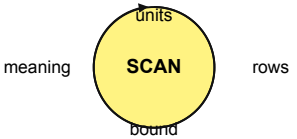
Prompt. Send five units from a source through two intermediate routes to a sink.

1. Define one nonnegative variable for each directed arc.
2. At every intermediate node: total inflow = total outflow.



sum of node balances = 0

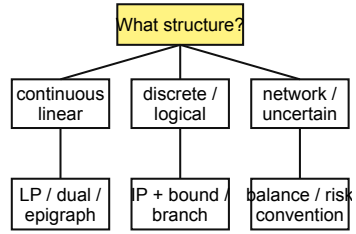
Network audit: source, transshipment and sink balances. *[ch. 9 example]*



Final audit loop: units → rows → bound → meaning. *[study strategy]*

24 · Method chooser CHS 01–10

IF YOU SEE X	DO Y	CERTIFICATE / CHECK
Two-variable LP	Find feasible vertices	Vertex values or dual bound
Candidate optimum	Test feasibility first	Independent bound
Blend/average claim	Use $\theta x^1+(1-\theta)x^2$	$0\le\theta\le1$; all rows preserved
Max of affine terms	Epigraph t above every term	Objective makes t tight
Resource price	Read dual multiplier locally	Basis/range unchanged
2×2 zero-sum game	Write response lines	Equalise; check endpoints
Whole-number choice	Integer/binary domain	Relaxation bound + incumbent
Routing/assignment	Directed arcs + balances	Node conservation/cut
Uncertain timing	Separate first stage/recourse	No advance knowledge
Tail loss	Sort loss distribution	Confidence + atom convention



Revision decision tree: identify structure before selecting machinery. *[topic maps]*

25 · Classic traps ALL CHAPTERS

1. Variables without units or time/entity indices.
2. Parameters accidentally treated as choices.
3. Capacity and requirement inequality directions swapped.
4. Objective value compared before feasibility is established.
5. Matrix product used despite incompatible inner dimensions.
6. Boundary intersection accepted without checking all rows.

26 · Fast glossary COURSE GLOSSARY

FEASIBLE	all rows + domains satisfied
EXTREME POINT	not a proper convex blend of two distinct feasible points
DUAL	companion bounding model
VALID INEQUALITY	holds for every feasible integer solution
MINIMUM CUT	source–sink partition giving a flow bound
CVAR	average loss in stated worst tail

Check your exam instructions on the LMS for what you may bring; use this sheet to revise and self-test.
Semester 2 formal examination period, from 16 November 2026 (the Exams Office publishes the exact date).