ECON1003 Quantitative Methods in Economics

UNIVERSITY OF SYDNEY · SCHOOL OF ECONOMICS

EXAM REVISION
Sem 1 2026 · **SIDE 1 OF 2**
Midterm 40% + Final 50% · formula sheet provided

SIDE 1/2 LINES → CALCULUS · Straight lines & elasticity · Quadratics/exponentials/logs · Financial maths · Differentiation & MR=MC

MATHS, NOT STATS

Compiled by **AskSia** · mapped to the ECON1003 syllabus · asksia.ai/cheatsheet/usyd-econ1003

0 · How to Use This — READ FIRST

★ ECON1003 is maths for economics — NOT statistics. No probability, distributions or hypothesis tests. Seven topics: lines · non-linear functions · financial maths · differentiation · several variables · integration · linear algebra (Bradley Chs 2,4,5,6,7,8,9).

Two exams: Midterm 40% (Wks 1–4, up to applications of differentiation — Side 1) · Final 50% (cumulative, all 7 — both sides). 4 quizzes = 10%. A **formula sheet is provided**; memorise the rest (power/product/chain rules, substitution, Lagrangian conditions, Cramer, inverse).

SIA → *Method marks matter – show every step; partial credit is given. Round only the FINAL answer to the stated decimal places.*

1 · The Straight Line — CH2

LINE

$y = mx + c$ m = slope, c = y-intercept

$m = (y_2-y_1)/(x_2-x_1)$ = rise/run

Slope m: "x up 1 ⇒ y changes by m." Intercept c: y when x=0. Horizontal line y=k; vertical x=k (undefined slope).

WORKED · LINE THRU (2,1), M=1

$1 = 1\cdot 2 + c \Rightarrow c = -1 \Rightarrow$ **$y = x - 1$**

TRAP → *Non-standard form (e.g. $2y+4x-4=0$) must be rearranged to $y=mx+c$ before reading m,c: ⇒ $y = -2x + 2$.*

2 · Demand & Supply — CH2

Demand Q=f(P), e.g. Q=200−2P (−2: a $1 rise drops Qd by 2). **Inverse demand** P=f(Q): P=100−0.5Q. The two slopes are reciprocals, not equal.

COST · REVENUE · PROFIT

$TC = FC + VC$ (e.g. $800 + 1.5Q$)

$R = P\cdot Q$ Profit = R − TC

Break-even: set Profit = 0, solve Q

WORKED · TC=800+1.5Q, P=3.5

Profit $= 3.5Q - (800+1.5Q) = 2Q-800 = 0 \Rightarrow$ break-even Q = **400**

TRAP → *Fixed cost is the intercept of TC, never part of marginal/variable cost. Read whether Q=f(P) or P=f(Q) is wanted.*

3 · Budget Line — CH2

TWO GOODS, INCOME M

$m = p_1x_1 + p_2x_2$

$x_2 = m/p_2 - (p_1/p_2)x_1$

Slope $-p_1/p_2$ = relative price; intercept m/p_2 = max of good 2. Price rise pivots (slope); income change shifts parallel (intercept only) — don't confuse.

WORKED · M=120, P₁=4, P₂=6

$x_2 = 20 - (2/3)x_1$ (max 30 of good 1, 20 of good 2)

4 · Linear Elasticity — CH2

POINT ELASTICITY

$E = \%\Delta Q / \%\Delta P = (dQ/dP)\cdot(P/Q)$

demand $P=a-bQ$: $Ed = (-1/b)\cdot(P/Q)$

supply $P=c+dQ$: $Es = (1/d)\cdot(P/Q)$

Demand: **elastic** Ed<−1 · **inelastic** −1<Ed<0 · **unit** Ed=−1.

WORKED · P=2400−0.5Q AT P=1800

$Q=1200 \Rightarrow Ed=(-1/0.5)(1800/1200)=$ **−3** (1% price ↑ ⇒ 3% quantity ↓, elastic)

TRAP → *Elasticity is NOT the slope – it varies along a straight line: elastic at high P, inelastic at low P, unit at the midpoint.*

5 · Quadratics — CH4

SOLVE AX²+BX+C=0

$x = (-b \pm \sqrt{b^2-4ac}) / (2a)$

discriminant $\Delta = b^2-4ac$

Δ>0 two real roots · Δ=0 one · Δ<0 no real roots (write "no real solution"). Parabola: one turning point (min if a>0, max if a<0); axis midway between roots; y-int = c.

WORKED · Y=2X²−7X−9

roots $x=-1,\ 4.5 \Rightarrow$ axis $x=1.75$

sub back ⇒ turning-point $y = -15.125$

DISCRIMINANT CHECK · X²+X+1

$\Delta = 1-4 = -3 < 0 \Rightarrow$ **no real roots**

Economic use: profit = quadratic in Q ⇒ the vertex gives the profit-maximising Q directly.

6 · Transformations — CH4

MOVE	EFFECT
f(x)+c	up c · f(x)−d down d
f(x−c)	RIGHT c · f(x+d) LEFT d
−f(x)	reflect in x-axis
f(−x)	reflect in y-axis

Combine in order: e.g. $y=-(x-3)^2+5$ is x^2 reflected, shifted right 3, up 5 (vertex (3,5), opens down).

TRAP → *Horizontal shifts go the "wrong" way – $(x-2)^2$ moves right, not left.*

7 · Exponentials & e — CH4

INDEX RULES · Y=Aˣ

$a^m\cdot a^n = a^{m+n}$ $a^m/a^n = a^{m-n}$

$(a^m)^k = a^{mk}$ $a^0=1$ $a^{-n}=1/a^n$

$e \approx 2.71828$

Solve by matching base, then equate exponents: $2^x = 1/16 = 2^{-4} \Rightarrow x = -4$. Growth: $P = P_0e^{rt}$.

WORKED · 3^{2X} = 81

$81 = 3^4 \Rightarrow 2x = 4 \Rightarrow x = 2$

TRAP → *Can't match the base? Take logs of both sides and use $\log(a^b)=b\cdot\log a$ to drop the exponent down.*

8 · Logarithms — CH4

log_a(b) = power on a giving b. $\log=\log_{10}$, $\ln=\log_e$.

LOG RULES

$\log(a)+\log(b) = \log(ab)$

$\log(a)-\log(b) = \log(a/b)$

$\log(a^b) = b\cdot\log(a)$

change base: $\log_c(a) = \log(a)/\log(c)$

WORKED · 1750 = 753E^{0.03T}

$\ln(1750/753) = 0.03t \Rightarrow t \approx$ **28**

TRAP → *$\ln(A+B) \neq \ln A + \ln B$. Rules apply only to ×/÷ inside one log; argument must be >0.*

9 · Sequences & Series — CH5

N-TH TERM

arithmetic: $a_n = a + (n-1)d$

geometric: $a_n = a\cdot r^{n-1}$

ARITHMETIC SUM (+D)

$S_n = (n/2)(2a + (n-1)d)$

GEOMETRIC SUM (×R)

$S_n = a(1-r^n)/(1-r)$

infinite ($|r|<1$): $S = a/(1-r)$

WORKED · 2+6+18+… (8 TERMS)

$a=2,\ r=3 \Rightarrow S_8 = 2(3^8-1)/2 =$ **6560**

TRAP → *Want the n-th term or the sum? And the infinite sum needs $|r|<1$.*

10 · Interest — CH5 · ON SHEET

GROWTH OF P₀

simple: $P_t = P_0(1+it)$

compound: $P_t = P_0(1+i)^t$

m/yr: $P_t = P_0(1+i/m)^{mt}$

continuous: $P_t = P_0e^{it}$

PV: $P_0 = P_t/(1+i)^t$

WORKED · 10000→20000 IN 6YR

$2=(1+i)^6 \Rightarrow i = 2^{1/6}-1 \approx$ **0.122**

TRAP → *With m periods/yr you must divide rate (i/m) AND multiply periods (mt) – the classic slip drops the mt.*

11 · Depreciation & NPV — CH5

Straight-line: subtract a fixed amount/yr. **Reducing-balance:** $A_t = A_0(1-i)^t$; total depr = $A_0 - A_t$.

NET PRESENT VALUE

$NPV = \Sigma\ cashflow_t/(1+i)^t$

outlay at t=0 enters NEGATIVE

NPV > 0 ⇒ project beats the discount rate.

WORKED · $1000, QUARTERLY, 8%, 2YR

$P = 1000(1+0.08/4)^{4\cdot 2} = 1000(1.02)^8 \approx$ **$1171.66**

WORKED NPV · −1000 NOW, +600/YR ×2, I=10%

$-1000 + 600/1.1 + 600/1.21 \approx -1000 + 545.5 + 495.9 =$ **+41.3** ⇒ accept

12 · Annuities & Loans — CH5 · ON SHEET

FUTURE VALUE (DEPOSIT A₀)

$V_t = A_0\cdot[(1+i/m)^{tm} - 1]/(i/m)$

PRESENT VALUE / LOAN L

$L = A_0\cdot[1 - (1+i/m)^{-mt}]/(i/m)$

solve for A_0 = each payment

Total interest = (payment × no. payments) − L.

WORKED · LOAN $20000, 6%/YR, 5YR ANN.

$L = A_0\cdot[1-(1.06)^{-5}]/0.06$

$20000 = A_0\cdot 4.2124 \Rightarrow A_0 \approx$ **$4747.93/yr**

TRAP → *FV uses $(\ldots)^{tm} - 1$; PV/loan uses $1 - (\ldots)^{-tm}$ (negative exp). Match m to payment frequency (monthly m=12).*

13 · The Derivative — CH6

DEFINITION (NOT EXAMINED TO COMPUTE)

$f'(x) = \lim_{k\to 0} [f(x+k)-f(x)]/k$

= instantaneous rate of change = slope of the tangent. **Notation:** f'(x), dy/dx, df/dx all mean the same thing.

14 · Differentiation Rules — CH6 · MEMORISE

CORE RULES

power: $x^n \to n\cdot x^{n-1}$ const → 0

$[Kf]' = Kf'$ $[f+g]' = f'+g'$

$a^x \to a^x\cdot\ln a$ $e^x \to e^x$

$\log_a x \to 1/(x \ln a)$ $\ln x \to 1/x$

chain: $dy/dx = (dy/du)(du/dx)$

product: $[fg]' = f'g + g'f$

quotient: $[f/g]' = (f'g - g'f)/g^2$

REWRITE-FIRST WORKED · Y = 3/X²

$= 3x^{-2} \Rightarrow y' = -6x^{-3} = -6/x^3$

TRAP → *Rewrite roots/fractions as powers first ($\sqrt{x}=x^{0.5}$, $1/x=x^{-1}$). Quotient order is f'g − g'f. Chain: ×inner derivative.*

15 · Curve Shape — CH6

SIGN	MEANING
f'>0 / <0 / =0	↑ / ↓ / stationary
f''>0	concave up (convex)
f''<0	concave down (concave)

Turning pt: f'=0 AND slope changes sign. **Inflection:** f''=0 AND concavity changes.

WORKED · Y = X³ − 3X²

$f'=3x^2-6x=0 \Rightarrow x=0,2$ (turning pts)

$f''=6x-6=0 \Rightarrow x=1$ (inflection)

$f''(0)=-6<0$ MAX · $f''(2)=6>0$ MIN

Higher derivatives: f''' and beyond just differentiate again; f'' is the one with economic meaning (curvature, diminishing returns). On a TC curve f''>0 means MC is rising.

TRAP → *f'=0 ≠ turning point (e.g. $y=x^3$ at 0); always check the sign actually changes.*

16 · Optimisation — CH6

2ND-DERIVATIVE TEST (F'=0)

$f''<0 \Rightarrow$ local MAX

$f''>0 \Rightarrow$ local MIN

$f''=0 \Rightarrow$ inconclusive (use f' sign test)

Global: evaluate f at every stationary point + compare.

WORKED MIN · F=X²−6X+5

$f'=2x-6=0 \Rightarrow x=3$ · $f''=2>0 \Rightarrow$ MIN

$f(3) = -4$

WORKED MAX · F=−X²+4X

$f'=-2x+4=0 \Rightarrow x=2$ · $f''=-2<0 \Rightarrow$ MAX

$f(2) = 4$

17 · Economic Applications — CH6 · EXAM FAVE

MARGINALS

$MR = dTR/dQ$ $MC = dTC/dQ$

revenue max: MR = 0

profit max: **MR = MC**

Non-linear elasticity: $Ed = (dQ/dP)\cdot(P/Q)$ with calculus for dQ/dP.

WORKED · P=100−Q, TC=Q²

$TR=PQ=100Q-Q^2 \Rightarrow MR=100-2Q$

$MC=2Q$ · $MR=MC \Rightarrow Q=$**25**, P=75

check: revenue max (MR=0) at Q=50 — different!

TRAP → *To get MR, first write TR=P·Q with P in terms of Q, THEN differentiate. Never differentiate price directly.*

17b · Worked Derivatives — CH6 · THE RULES IN ACTION

CHAIN · Y = (3X²+1)⁵

$= 5(3x^2+1)^4\cdot 6x = 30x(3x^2+1)^4$

PRODUCT · Y = X²Eˣ

$= 2x\cdot e^x + x^2\cdot e^x = e^x(x^2+2x)$

QUOTIENT · Y = (X+1)/(X−1)

$= [(x-1)-(x+1)]/(x-1)^2 = -2/(x-1)^2$

18 · Midterm Blueprint — 40% · 60 MIN

Held 19 Apr; covers Wks 1–4 to applications of differentiation: lines, elasticity, quadratics, logs/exponentials, financial maths, differentiation + MR/MC (Side 1). 60 min, formula sheet provided.

Drill: rearrange to y=mx+c · pick FV vs PV annuity · MR=MC profit · 2nd-deriv test. Show work; round at the end.

Bring nothing you don't understand — the formula sheet gives you the quadratic formula, interest/annuity/NPV, elasticity, the a^x and log derivatives and the quotient rule, but not the power/product/chain rules. Those you must know cold.

★ Formula Belt · Side 1 — MEMORISE COLD

LINES & ELASTICITY

$y = mx + c$ · $m = \Delta y/\Delta x$

Profit = PQ − TC; break-even Profit=0

budget $x_2 = m/p_2 - (p_1/p_2)x_1$

$Ed = (dQ/dP)(P/Q)$

NON-LINEAR

$x = (-b\pm\sqrt{b^2-4ac})/(2a)$

$\Delta = b^2-4ac$

$\log(ab)=\log a+\log b$ · $\log(a^b)=b \log a$

FINANCE

$P_t = P_0(1+i)^t$ · $= P_0(1+i/m)^{mt}$

continuous $P_t = P_0e^{it}$

$NPV = \Sigma\ CF_t/(1+i)^t$

annuity PV ∝ $1-(1+i/m)^{-tm}$

CALCULUS

$x^n\to nx^{n-1}$ · $e^x\to e^x$ · $\ln x\to 1/x$

chain $(dy/du)(du/dx)$ · $a^x\to a^x\ln a$

$[fg]'=f'g+g'f$ · $[f/g]'=(f'g-g'f)/g^2$

2nd-deriv: f''<0 max, f''>0 min

MR=MC at profit max · MR=0 at rev max

★ Exam Discipline — DON'T LOSE EASY MARKS

- **Rearrange first** — get y=mx+c or P=f(Q) before reading anything off
- **Units & signs** — Ed is negative; FC sits in the intercept
- **m-periods** — divide rate, multiply exponent, both
- **Δ<0** — say "no real solution," don't force it
- **+C** habit even on Side 1 (it matters on Side 2)
- **Round last** — to the stated decimals only

SIA → *The single most-tested move on this side is profit maximisation: build TR(Q), differentiate to MR, set MR=MC, solve, back-out P. Drill it until automatic.*

★ Quick Self-Check — CAN YOU DO THESE?

- Rearrange 3y−6x+9=0 → y=2x−3
- Ed of P=50−2Q at Q=10 (P=30) = −1.5
- $2^x = 32 \Rightarrow x = 5$
- $5000 at 8% cts for 3yr = $5000e^{0.24} \approx$ $6356
- $d/dx[x^2\ln x] = 2x \ln x + x$
- Roots of $x^2-5x+6 = 0 \to x = 2, 3$
- Break-even: TC=100+4Q, P=9 → Q = 20
- Turning pt of $y=x^2-8x+1 \to (4, -15)$
- $\int(6x^2+2)\,dx \to 2x^3 + 2x + C$

★ Quiz & Logistics — 10% · DON'T LOSE IT

4 online quizzes (10% total), 1 week each, NO extensions — do them early. Q&A via **Ed**, not email. Textbook: Bradley, *Essential Mathematics for Economics & Business*, 4th ed. (Chs 2,4,5,6,7,8,9). The unit is maths *in the service of economics* — every technique gets an economic reading (slope = marginal effect, λ = shadow price, integral = surplus).

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SIDE 2/2 SEVERAL VARIABLES → MATRICES · Partials · MPL/MPK · MRTS · MRS · the LAGRANGIAN · Integration & surplus · Linear algebra · Cramer · inverse

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19 · Partial Derivatives (CH7)

Method: for $\partial f/\partial x$, treat every other variable as a constant. 2nd-order: f_xx, f_yy; cross-partials f_xy = f_yx (mixed partials equal).

WORKED · $z = x^2y + 2y + 4$

$\partial z/\partial x = 2xy \quad \partial z/\partial y = x^2 + 2$

TOTAL DIFFERENTIAL

$dz = (\partial z/\partial x)dx + (\partial z/\partial y)dy$

Use the differential to approximate a small change: if x rises by dx and y by dy, z changes by about dz — handy for "% change in output" questions.

TRAP → *Track which variable is "the constant" – lone variables & constant-multiple terms vanish or simplify depending on it.*

20 · Production · MPL/MPK (CH7)

Q = f(L,K). MPL = $\partial Q/\partial L$, MPK = $\partial Q/\partial K$. Diminishing returns: Q_LL < 0 (MPL falls as L↑). Cobb–Douglas e.g. Q = 10L^{0.7}K^{0.3}.

WORKED · $Q = 10L^{0.7}K^{0.3}$

$MPL = 7L^{-0.3}K^{0.3} \quad MPK = 3L^{0.7}K^{-0.7}$

ISOQUANT & MRTS

isoquant = all (L,K) with same Q

MRTS = dK/dL = -MPL/MPK

21 · Utility · MRS (CH7)

U(x,y); MUx = $\partial U/\partial x$, MUy = $\partial U/\partial y$. Indifference curve = all (x,y) with same U.

MARGINAL RATE OF SUBSTITUTION

MRS = dy/dx = -MUy/MUx

WORKED MRS · $U = x^{0.5}y^{0.5}$

$MUx = 0.5x^{-0.5}y^{0.5}$, $MUy = 0.5x^{0.5}y^{-0.5}$

MRS = -MUy/MUx = -x/y

PARTIAL ELASTICITIES

price Ed = $(\partial Q/\partial P\_A)(P\_A/Q)$

income E_Y = $(\partial Q/\partial Y)(Y/Q)$

cross Ec = $(\partial Q/\partial P\_B)(P\_B/Q)$

Signs read the goods: Ec>0 substitutes, Ec<0 complements; E_Y>0 normal, E_Y<0 inferior.

22 · Unconstrained Optim. (CH7)

2 VARIABLES

set $\partial f/\partial x = 0$ AND $\partial f/\partial y = 0$

solve simultaneously, evaluate f

(Full 2nd-order test mentioned, not examined — assume the extremum exists if asked.)

WORKED · $F = x^2 + y^2 - 4x - 6y$

2x-4=0 ⇒ x=2 · 2y-6=0 ⇒ y=3

min at (2,3), f = -13

TRAP → *The two first-order conditions are simultaneous – substitute one into the other if they're coupled.*

23 · The Lagrangian (CH7 · LECTURE NOTES)

Max f(x,y) s.t. equality h(x,y)-c=0 & inequality g(x,y)-b ≤ 0 constraints. Each constraint gets its own multiplier λ.

BUILD IT (MINUS EACH CONSTRAINT)

$L = f - \lambda_1(h_1-c_1) - \lambda_2(g_1-b_1) - \dots$

CONDITIONS — ALL MUST HOLD

1. $\partial L/\partial x = 0$, $\partial L/\partial y = 0$
2. all constraints hold
3. inequality multipliers λ ≥ 0
4. compl. slackness: $\lambda_j(g_j-b_j) = 0$

Method: write L → list conditions → solve case-by-case (x=0? y=0? both >0?) → evaluate f at each → pick best. **Min f** = max(−f).

CASE LOGIC FOR ONE INEQUALITY G−B≤0

Case A: λ=0 (slack) → ignore constraint, check g≤b

Case B: g=b (binds) → solve with λ≥0

TRAP → *Complementary slackness spawns cases – students solve only the interior & forget the x=0, y=0 corners. Reject any candidate forcing an inequality λ negative.*

23b · Worked · Utility Max (THE WHOLE MOVE)

MAX U=XY S.T. 2X+Y=10

L = xy - λ(2x+y-10)

L_x: y - 2λ = 0 ⇒ y = 2λ

L_y: x - λ = 0 ⇒ x = λ

⇒ y = 2x; sub: 2x+2x=10 ⇒ x=2.5

x=2.5, y=5, λ=2.5, U=12.5

At the optimum MRS = price ratio: MUx/MUy = y/x = 2 = p_1/p_2. ✓

Cost-min mirrors it: min C s.t. Q̄ = f(L,K) gives the tangency MRTS = w/r (input-price ratio).

24 · The Multiplier λ (CH7 · SHADOW PRICE)

λ = shadow price: how much the optimal objective changes if you relax that constraint by one unit.

In utility-max s.t. budget: λ = **marginal utility of income** (above, λ=2.5 ⇒ +$1 income raises U by ≈2.5). In cost-min s.t. output: λ = **marginal cost** of one more unit. Uses: utility maximisation & cost minimisation.

SIA → *If asked "what does λ mean?", answer in units of the objective per unit of the constraint – that one sentence is worth marks.*

24b · Lagrangian Checklist (BEFORE YOU STOP)

- All ∂L/∂(var) = 0; constraints checked
- Inequality λ's all ≥ 0
- Complementary slackness on each ≤
- Corner cases (x=0, y=0) evaluated
- f compared across ALL candidates

25 · The Integral (CH8)

ANTI-DERIVATIVE

∫f(x)dx = g(x) + C, where g'(x)=f(x)

ALWAYS add + C (indefinite)

RULES

$\int x^n\,dx = x^{n+1}/(n+1) + C \quad (n \neq -1)$

$\int x^{-1}\,dx = \ln(x) + C$ (the n=-1 case)

$\int K\,dx = Kx + C \cdot \int e^x\,dx = e^x + C$

TRAP → *Power rule breaks at n=−1 (÷0) – that case is the log. Dropping +C is fatal once you solve for C in an ODE.*

26 · Substitution (CH8)

Reverse of the chain rule: set u = inner fn, find du, rewrite all in u, integrate, sub back.

WORKED · $\int(5x-2)^{10}\,dx$

u=5x-2, dx=du/5

⇒ $(5x-2)^{11}/55 + C$

WORKED · $\int 2x\cdot e^{x^2}\,dx$

$u=x^2$ ⇒ du=2x dx ⇒ $\int e^u\,du = e^{x^2}+C$

TRAP → *The leftover x's must all cancel via du – if any remain, substitution isn't the right tool.*

27 · By Parts (CH8 · ON SHEET)

LECTURER'S FORM

∫f g dx = f·∫g dx - ∫f'·(∫g dx) dx

WORKED · ∫LN X DX

f=ln x, g=1 ⇒ x ln x - x + C

WORKED · $\int x e^x\,dx$

$f=x$, $g=e^x$ ⇒ $x e^x - \int e^x\,dx = e^x(x-1)+C$

TRAP → *Pick f (to differentiate) so f' simplifies – logs & powers of x are good f.*

28 · Definite Integral & Area (CH8)

NET SIGNED AREA

$\int_a^b f\,dx = F(b) - F(a)$

between curves: $\int_a^b [f - g]\,dx$ (f above g)

WORKED · $\int_1^3(2x+1)\,dx$

$= [x^2+x]_1^3 = (9+3) - (1+1) = 10$

TRAP → *Gives NET area – below-axis counts negative ($\int_{-3}^{3}x\,dx = 0$). For "total area" split at crossings.*

28b · TC from MC (CH8 · ECON USE)

RECOVER TOTAL COST

TC = ∫ MC dQ + FC

the constant of integration = fixed cost

WORKED · $MC = 3Q^2+2$, FC=50

$TC = Q^3 + 2Q + 50$

29 · Surplus & ODEs (CH8)

CONSUMER / PRODUCER SURPLUS

$CS = \int_0^{Q^*} [\text{demand}(Q) - P^*]\,dQ$

$PS = \int_0^{Q^*} [P^* - \text{supply}(Q)]\,dQ$

Find equilibrium Q* (from P*) first — it's the upper limit.

WORKED CS · D: P=20−Q, P*=8

Q*=12 ⇒ $CS = \int_0^{12}(20-Q-8)\,dQ$

$= [12Q - Q^2/2]_0^{12} = 144 - 72 = 72$

WORKED PS · S: P=2+Q, P*=8

Q*=6 ⇒ $PS = \int_0^6(8-2-Q)\,dQ$

$= [6Q - Q^2/2]_0^6 = 36 - 18 = 18$

DIFFERENTIAL EQUATIONS

dy/dx=f(x): integrate, use a point to fix C

separable dy/dx=f(x)g(y): collect y left, x right, integrate both

Economic uses: TC = ∫MC dQ; accumulated quantity from a rate.

WORKED SEPARABLE · DY/DX = XY

∫(1/y)dy = ∫x dx

ln y = $x^2/2$ + C ⇒ y = $Ae^{x^2/2}$

The classic econ ODE dP/dt = kP separates to $P = P_0e^{kt}$ – the same exponential growth seen in continuous interest.

TRAP → *Keep + C until the initial condition is applied; on separable eqns remember the ln/exp step when re-isolating y.*

30 · Matrices · Basics (CH9)

Dimension = rows × cols; equal iff same dim & all entries match. **Transpose A^T**: swap rows↔cols (a×b → b×a). Special: null, square, identity I (1's on diagonal).

ARITHMETIC

add/subtract: element-wise, SAME dim only

scalar: × every entry

A(m×n)·B(n×k) = m×k; cols A = rows B

WORKED · PRODUCT (ENTRY = ROW·COL)

[[1,2],[3,4]]·[[5],[6]] = [[17],[39]]

(1·5+2·6=17 · 3·5+4·6=39)

Identity I acts like 1: AI = IA = A. Transposing reverses a product: $(AB)^T = B^TA^T$.

TRAP → *Multiplication is not commutative – AB ≠ BA (one may not even be defined). Check dims before multiplying.*

30b · Economic Use (CH9 · WHY BOTHER)

A market or input–output model is a **system of linear equations**; in matrix form AX = B. Solving it (Gaussian, Cramer, or the inverse) gives equilibrium prices/quantities all at once. The determinant flags whether a unique equilibrium even exists.

31 · Elimination (CH9)

Strip variables → augmented matrix [A|b]. **3 row ops** (none change the solution): swap rows · ×non-zero · add a multiple of one row to another.

Gaussian → row-echelon (more leading zeros each row down), then back-substitute. **Gauss–Jordan** → reduced row-echelon (1's on diagonal, 0's elsewhere) ⇒ read the solution off directly.

OUTCOMES FROM THE ECHELON FORM

full diagonal → unique solution

0 = 0 row → infinitely many

0 = nonzero row → no solution

TRAP → *Arithmetic slips in row ops are the #1 error – keep the RHS column attached and operate on the whole row.*

32 · Determinants (CH9 · 3×3 ON SHEET)

2×2

det[[a,b],[c,d]] = ad - bc

3×3 · COFACTOR ON TOP ROW (+ − +)

a·|minor_a| - b·|minor_b| + c·|minor_c|

WORKED 3×3 · DIAG-HEAVY

det[[1,2,0],[0,3,1],[0,0,4]] (triangular)

= product of diagonal = 1·3·4 = 12

TRAP → *The alternating + − + signs on the 3×3 are routinely forgotten. (Triangular det = product of the diagonal – a fast check.)*

33 · Cramer's Rule (CH9 · MEMORISE)

SOLVE AX = B

$x = \det(B_x)/\det(A) \cdot y = \det(B_y)/\det(A)$

B_{var} = A with that column replaced by b

requires det(A) ≠ 0

WORKED · 2X+Y=5, X+3Y=10

det A = 2·3-1·1 = 5

x = det[[5,1],[10,3]]/5 = (15-10)/5 = 1

y = det[[2,5],[1,10]]/5 = (20-5)/5 = 3

check: 2(1)+3=5 ✓

TRAP → *Replace the correct column per variable; the method fails entirely when det(A)=0 (no unique solution).*

33b · Which Method? (CH9 · PICK FAST)

- **One unknown wanted** → Cramer (fewer dets)
- **Same A, many b's** → find A^{-1} once
- **Big / numeric system** → Gauss–Jordan

34 · The Inverse Matrix (CH9 · MEMORISE)

DEFN — EXISTS IFF DET(A) ≠ 0

$A\cdot A^{-1} = A^{-1}\cdot A = I$

COFACTOR METHOD

$A^{-1} = C^T / \det(A)$

C = cofactors (minor × +-+ sign)

2×2 SHORTCUT

A^{-1} = (1/(ad-bc))·[[d,-b],[-c,a]]

WORKED · A = [[2,1],[1,1]]

det = 2-1 = 1 ⇒ A^{-1} = [[1,-1],[-1,2]]

SOLVE A SYSTEM

AX = B ⇒ X = A^{-1}B

TRAP → *Cofactor method: don't forget to TRANSPOSE (C') and divide by det(A). If det(A)=0 there is no inverse.*

★ Formula Belt · Side 2 (MEMORISE COLD)

SEVERAL VARIABLES

MPL=∂Q/∂L · MPK=∂Q/∂K

MRTS=-MPL/MPK · MRS=-MUy/MUx

L = f - λ(constraint) · λ = shadow price

compl. slackness λ(g-b)=0

INTEGRATION

$\int x^n = x^{n+1}/(n+1)+C$ (n≠-1) · $\int x^{-1} = \ln x + C$

by parts: f∫g - ∫f'(∫g)

$CS = \int_0^{Q^*}(D-P^*)$ · $PS = \int_0^{Q^*}(P^*-S)$

LINEAR ALGEBRA

det 2×2 = ad-bc

Cramer $x_i = \det(B_i)/\det(A)$

$A^{-1} = C^T/\det(A)$ · X = A^{-1}B

ON THE SHEET	YOU MEMORISE
3×3 determinant	Cramer's rule
a^x, log_a, quotient rule	inverse methods
by-parts formula	Lagrangian conditions

★ Final Exam Discipline (ALL 7 TOPICS)

- **Cumulative** — Side 1 maths returns; redo profit-max & finance
- **Lagrangian** — list ALL 4 conditions; check corners
- **Integration** — n=−1 → ln; +C; find Q* before surplus
- **Matrices** — dims first; det≠0 before Cramer/inverse; transpose cofactors
- **Surplus** — equilibrium Q* is the upper limit, not P*
- **Show work** — partial credit; round only at the end

SIA → *The big Side-2 earners are the Lagrangian (with shadow-price interpretation) and a full matrix solve (Cramer or inverse) – rehearse one of each end-to-end the night before.*