MATH1131 Mathematics 1A · Rules, Methods & Checks

UNSW SYDNEY · TERM 3, 2026 · EXAM REVISION SHEET

Full guide + ask Sia about MATH1131

SIDE 1 · RECALL + CHOOSE — **Final exam:** 50% · 2 hours · in person on campus — **DEFINITIONS → METHOD → HYPOTHESES → CHECK**

0 · Exam map

COURSE.JSON · T3 2026

M10 T5 L1 10 A10 L2 15 FINAL 50%

Weekly Lessons: Möbius	10%
Weekly Lessons: Tutorial Participation	5%
Lab test 1	10%
Assignment	10%
Lab test 2	15%
Final exam	**50%**

Permitted materials: not confirmed for this offering in the course materials we hold — check the exam instructions on the LMS.

1 · What the final rewards

COURSE OVERVIEW + FAQ

PROMPT	VISIBLE WORKING
Compute	formula → substitution → exact result
Show / prove	name result; verify every hypothesis; conclude
Solve	method steps + all cases + check in original
Sketch	domain, intercepts, asymptotes, stationary points, labels

UNIVERSAL ANSWER PATH

```
diagnose object → choose result →
state conditions → calculate →
interpret → independent check
```

SIA → *Keep exact fractions and surds until the last line. A substitution, dot-product zero, determinant property or sketch is a fast error detector.*



Line source-redraw: one fixed point plus every real multiple of a direction.

2 · Lines & planes

C1

```
line: x = a + td, t∈ℝ
plane: x = a + su + tv, s,t∈ℝ; u,v
non-parallel
point-normal: n·(x-a)=0
```

Membership: one parameter value must satisfy every coordinate. Two space lines: directions multiples → parallel; otherwise solve both parameters—no solution means *skew*. Plane from three points: u=B−A, v=C−A, then n=u×v and verify all three points.

TRANSLATION AND CLASSIFICATION

two points P,Q	d=Q−P; x=P+td
symmetric line	$(x-a_1)/d_1=(y-a_2)/d_2=(z-a_3)/d_3=t$; handle a zero direction coordinate separately
two plane directions	non-parallel u,v span a plane; u×v supplies a normal
Cartesian plane	Ax+By+Cz=D has normal (A,B,C)

3 · Vector chooser

C2 · FAQ

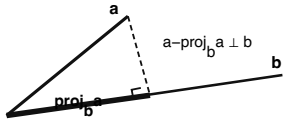
IF ASKED FOR	USE
length / distance	$\|a\|=\sqrt{(a\cdot a)}$
angle / perpendicular	$a\cdot b=\|a\|\|b\|\cos\theta$; $a\cdot b=0$
component along b	projection
normal / area in $\mathbb{R}^3$	cross product
volume / coplanarity	$[a\cdot(b\times c)]$; zero ⇔ coplanar

COORDINATE RULES

$$a\cdot b=\Sigma a_i b_i$$

$$\text{proj}_b a=(a\cdot b/\|b\|^2)b$$

$$a\times b=(a_2b_3-a_3b_2,\ a_3b_1-a_1b_3,\ a_1b_2-a_2b_1)$$



Projection source-redraw: subtract the shadow to obtain the shortest perpendicular component.

3b · Area, volume, distance

C2

```
parallelogram area = ‖a×b‖; triangle
= ½‖a×b‖
parallelepiped volume = |a·(b×c)|
dist(P, Ax+By+Cz=D)=|Ax₀+By₀+Cz₀-D|/
√(A²+B²+C²)
```

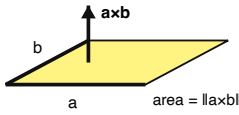
For a point-to-line distance, choose A on the line, form AP, subtract proj_dAP, then take the norm. Cross-product order changes sign, not magnitude; check $(a\times b)\cdot a=(a\times b)\cdot b=0$.

PROJECTION AND ANGLE CHECKS

```
scalar component of a along b = a·b/
‖b‖
vector component = proj_b a
perpendicular component = a-proj_b a
```

For the angle between non-zero vectors, compute $\cos\theta=(a\cdot b)/(\|a\|\|b\|)$ and use $\theta\in[0,\pi]$. For the acute angle between planes, use their normals and take the acute representative. An orthonormal set has unit lengths and pairwise zero dot products; expansion coefficients along its vectors are dot products.

QUICK VERDICT	TEST
orthogonal	$a\cdot b=0$
parallel	a=cb for some scalar c
coplanar triples	$a\cdot(b\times c)=0$
point H closest on line	H lies on line and PH·d=0



Cross-product source-redraw: direction is normal; length is parallelogram area.

4 · Complex numbers

C3–C4

VIEW	BEST FOR	RULE
z=a+bi	add, compare parts, divide	$i^2=-1$
z=a−bi	division, real tests	$z\bar{z}=\|z\|^2$
$re^{i\theta}$	products, powers, roots	angles add
Argand	regions, quadrant checks	$\|z-c\|$=distance

```
Re(a + bi)=a; Im(a + bi)=b not bi
|z|=√(a²+b²); |z/w|=|z|/|w|
z/w = z w̄ /(w w̄)
```

Region $|z-c|<r$ is the open disc centred at c; strict boundary dashed. Complex numbers themselves are not ordered; inequalities apply to real quantities such as modulus or parts.

4b · Polar, powers, roots

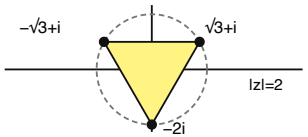
C4

$$z=r(\cos\theta+i\sin\theta)=re^{i\theta}$$

$$(re^{i\theta})^n=r^n e^{in\theta}$$

$$z^n=Re^{i\varphi}\Rightarrow z_k=R^{1/n}e^{i(\varphi+2k\pi)/n},\ k=0,\dots,n-1$$

Quadrant before arctan. Principal Arg z lies in $(-\pi,\pi]$. The n roots share one modulus, differ by $2\pi/n$ and form a regular polygon; plot them to expose omissions or duplicates.



Roots source-redraw: $z^3=8i$ gives angles π/6, 5π/6 and 3π/2.

4c · Polynomial route

C4 · FAQ

Factor theorem: P(α)=0 ⇔ (z−α) divides P. With real coefficients, a non-real root α forces $\bar{\alpha}$; combine the pair:

$$(z-a)(z-\bar{a})=z^2-2\text{Re}(a)z+|a|^2$$

State "coefficients are real" before using the conjugate-pair rule; divide out the quadratic, solve the remainder, then expand or substitute to check.

CONJUGATE IDENTITIES

```
z+z̄=2Re z · z-z̄=2i Im z
overline(z+w)=z̄+w̄ · overline(zw)=z̄w̄
overline(z/w)=z̄/w̄ (w≠0)
z real ⇔ z=z̄ ⇔ Im z=0
```

Square-root routine: set $(a+bi)^2=u+vi$, giving $a^2-b^2=u$ and $2ab=v$; add $a^2+b^2=|u+vi|$. Solve for a^2,b^2, then use the sign of ab to select the pair ±(a+bi). The two roots must be negatives.

OPERATION	GEOMETRY	CHECK
z+w	vector translation	parts add
$\bar{z}$	reflection in real axis	$\|\bar{z}\|=\|z\|$
iz	quarter-turn anticlockwise	modulus unchanged

5 · Gaussian elimination

C5

LEGAL MOVES

$$R_i\leftrightarrow R_j\ \cdot\ R_i\leftarrow cR_i\ (c\neq0)\ \cdot\ R_i\leftarrow R_i+cR_j$$

1. Choose the leftmost available non-zero pivot; swap if useful.
2. Clear below it; move right and down; stop at echelon form.
3. Read consistency and free variables, then back-substitute.
4. Write an infinite set as point + parameter directions; check original equations.

x	y	z	b	
1	2	−1	3	2 pivots
0	1	3	4	z free
0	0	0	0	1 param

Echelon source-redraw: harmless zero row; one non-leading unknown gives one parameter.

LAST FORM	VERDICT
[0 … 0 | c], c≠0	**no solution**
consistent + pivot every unknown column	unique
consistent + non-leading column	infinitely many

6 · Matrices & determinants

C6

```
(AB)ᵢⱼ = rowᵢ(A)·colⱼ(B); usually
AB≠BA
(AB)ᵀ=BᵀAᵀ; (AB)⁻¹=B⁻¹A⁻¹
A⁻¹ for 2×2 = (1/(ad-bc)) [[d,-b],
[-c,a]]
```

ROW ACTION	DET EFFECT
swap	changes sign
scale row by c	multiplies by c
add multiple of another	unchanged

```
det(Aᵀ)=det A · det(AB)=det A det B
det(cA)=cⁿdet A for n×n ·
det(A⁻¹)=1/det A
```

det A≠0 ⇔ A invertible ⇔ Ax=b has one solution for every b. If det A=0, the system has no unique solution; row reduction decides none versus infinitely many.

READING AND CHECKING MATRIX WORK

AB exists when columns(A)=rows(B); its size is rows(A)×columns(B). A zero product AB=0 does not force A=0 or B=0, and cancellation is unsafe unless the cancelled matrix is invertible. Transpose reverses multiplication order; inverse also reverses order.

Inverse by augmentation: row-reduce [A|I]. Every operation performed on A is also performed on I; reaching [I|B] proves $B=A^{-1}$. If the left block gains a zero row, A is singular and the process cannot reach I. Always multiply A by the candidate inverse.

DET TASK	FAST MOVE
many zeros	cofactor-expand along that row/column

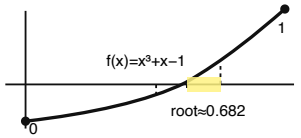
7 · Limits & continuity

C7

FORM	FIRST MOVE
polynomial ratio, x→∞	divide by highest denominator power
surd − surd / ∞−∞	multiply by conjugate
bounded oscillation × vanishing	pinch between common limits
piecewise at a	left limit = right limit = f(a)

```
continuity at a: lim_{x→a} f(x)=f(a)
two-sided limit exists ⇔ left limit
= right limit
```

IVT: f continuous on [a,b] and y lies between f(a),f(b) ⇒ some c∈[a,b] has f(c)=y. For a root, exhibit opposite signs. **Maximum–minimum theorem:** continuous on closed bounded [a,b] ⇒ global max and min are attained.



Bisection source-redraw: continuity plus a sign change traps the root; each midpoint halves the interval.

8 · Differentiation

C8 + C10

```
f'(a)=lim_{x→a}[f(x)-f(a)]/(x-a)
(uv)'=u'v+uv' · (u/v)'=(u'v-uv')/v²
(f∘g)'=f'(g)g' · linearisation
L(x)=f(a)+f'(a)(x-a)
```

Implicit: differentiate term-by-term, treat y=y(x), collect y'. **Piecewise differentiable:** first continuity, then equal one-sided derivatives. **Inverse:** if y=f(x), f'(x)≠0, then $(f^{-1})'(y)=1/f'(x)$.

RELATED-RATES ORDER
label changing quantities + units
write relation valid for all t
differentiate with respect to t
substitute instantaneous values last; state sign + units

STANDARD DERIVATIVE BANK

```
(xⁿ)'=nxⁿ⁻¹ · (eˣ)'=eˣ · (ln x)'=1/x
(sin x)'=cos x · (cos x)'=-sin x
(tan x)'=sec²x
(sin⁻¹x)'=1/√(1-x²)
(tan⁻¹x)'=1/(1+x²)
```

Derivative definition: at corners such as |x| at 0, compare one-sided difference quotients. Differentiability implies continuity, so failure of continuity ends the test; continuity alone does not guarantee a derivative.

SKETCH EVIDENCE	MEANING
f'>0 / f'<0	increasing / decreasing
f' changes + to −	local maximum
f' changes − to +	local minimum
f''>0 / f''<0	concave up / down

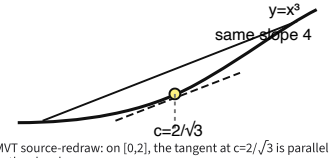
9 · MVT, Rolle, extremes

C9

MEAN VALUE THEOREM

```
f continuous on [a,b], differentiable
on (a,b)
⇒ ∃c∈(a,b): f'(c)=[f(b)-f(a)]/(b-a)
```

Rolle: add f(a)=f(b), giving f'(c)=0. Between every two zeros of f lies a zero of f'; if f' has at most k zeros, f has at most k+1. For inequalities: choose interval, quote MVT, bound f'(c) using c∈(a,b), conclude.



MVT source-redraw: on [0,2], the tangent at $c=2/\sqrt{3}$ is parallel to the chord.

9b · L'Hôpital & extremes

C9 FAQ

L'Hôpital only for 0/0 or ∞/∞ with differentiability nearby; differentiate numerator and denominator separately and re-check the form before repeating.

Rewrite 0·∞ as a quotient first; for 1^∞, 0^0, ∞^0, take ln, resolve, then exponentiate.

Global extrema on [a,b]: list stationary points, interior points where f' fails, and both endpoints; evaluate f at every candidate. If f''(c)=0, it is inconclusive—use the sign of f'.

10 · Integrals & logs

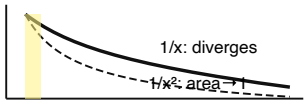
C11–C12

PATTERN	METHOD
f(g(x))g'(x)	substitution; change definite limits
product; one factor simplifies on d/dx	parts: $\int u\,dv=uv-\int v\,du$
∞ bound / unbounded integrand	write limit first
x variable in base and exponent	take ln, differentiate implicitly

$$\text{FTC I: } \frac{d}{dx}\int_a^{g(x)} f(t)\,dt=f(g(x))g'(x)$$

$$\text{FTC II: } \int_a^b f(x)\,dx=F(b)-F(a)$$

$$\int_1^\infty x^{-p}dx \text{ converges} \Leftrightarrow p>1$$



Improper-integral source-redraw: similar tails, opposite verdicts; define both by limits.

MATH1131 Self-test Drills, Traps & Fast Recall

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SIDE 2 · SELF-TEST — **Five worked paths:** cover the answer, recite the method, then check — **MIXED ALGEBRA + CALCULUS**

1 · Worked: line membership

C1 WORKED EXAMPLE

Q. Through P(2,1,−1), Q(5,−1,3); is R(11,−5,11) on the line?

1 · DIRECTION

$d=Q-P=(3,-2,4)$

2 · EQUATION

$x=(2,1,-1)+t(3,-2,4)$

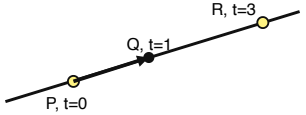
3 · FIX AND CHECK

$2+3t=11 \Rightarrow t=3$

$1-2(3)=-5;\ -1+4(3)=11$

all coordinates agree ⇒ R lies on the line.

One t must fit all coordinates. Different forced values mean the point is off the line.



Source-redraw: choose t from one coordinate; the others are checks, not new parameters.

1b · If you see X → do Y

C1–C2 FAQ

two points	subtract for direction; attach to either point
three points, plane	two directions → cross for normal → point-normal equation
two lines in $\mathbb{R}^3$	compare directions; then solve parameters
point to line	AP−proj$_d$AP, then norm
point to plane	project onto normal / coefficient formula

TRAP → *Parallel directions do not prove identical lines; also test whether a point of one lies on the other. Non-parallel with no common parameters means skew.*

CLOSED-BOOK-STYLE RECALL PROMPTS

Recite line form	fixed point + real multiple of non-zero direction
Recite plane form	fixed point + two real multiples of non-parallel directions
Normal from Cartesian plane	read coefficient vector
Plane from three points	cross two displacement vectors
Angle between planes	acute angle between normals

Distance check using the guide's numbers: for B(4,2,1), A(1,0,0), d=(1,1,0), AB=(3,2,1). proj$_d$AB=(5/2,5/2,0); the leftover (1/2,−1/2,1) dots to zero with d and has length $\sqrt{(3/2)}\approx1.225$. Cover this line and reproduce the projection denominator $\|d\|^2=2$.

2 · Worked: complex quotient

C3 WORKED EXAMPLE

Q. Write (5+i)/(2−3i) as a + bi and check its modulus.

CONJUGATE DENOMINATOR

$(5+i)(2+3i)/(4+9)$

$=(10+15i+2i+3i^2)/13$

$=(7+17i)/13$

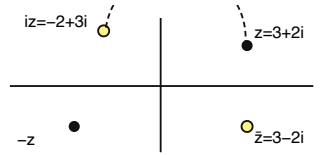
ANSWER + INDEPENDENT CHECK

$7/13 + (17/13)i$

$|5+i|/|2-3i| \models \sqrt{26}/\sqrt{13}=\sqrt{2}$

$|7+17i|/13=\sqrt{338}/13=\sqrt{2}$ ✓

If the moduli disagree, find the sign error.



Argand source-redraw: conjugate reflects; −z half-turns; multiplication by i rotates 90° anticlockwise.

2b · Polar judgement cards

C4

large integer power	Cartesian → polar → De Moivre → reduce angle → Cartesian
$z^n=w$	include $\phi+2k\pi$; k=0,…,n−1; draw polygon
real polynomial + one non-real root	include conjugate; form real quadratic
$\|z-c\|\le r$	circle centre c, radius r; shade correct side; strict boundary dashed

$(1+i)^{10}=(\sqrt{2}\ e^{i\pi/4})^{10}=32e^{i5\pi/2}=32i$

TRAP → *arctan(b/a) supplies a reference angle, not the quadrant. Plot the point before choosing Arg z.*

TWO RAPID ANSWER CHECKS

Square roots: for −5+12i, equations $a^2-b^2=-5$, $2ab=12$ and $a^2+b^2=13$ give $a^2=4$, $b^2=9$ and ab>0, so the roots are ±(2+3i). Squaring either root returns −5+12i.

Cube roots: $8i=8e^{i(\pi/2+2k\pi)}$. Hence $z=2e^{i(\pi/6+2k\pi/3)}$, k=0,1,2, giving $\sqrt{3}+i$, $-\sqrt{3}+i$, $-2i$. The arguments differ by 2π/3 and the three points form an equilateral triangle.

COMMON SLIP	REPAIR
Im z written with i	imaginary part is the real coefficient
one argument only	θ+2kπ before roots
missing root	list exactly n distinct k-values
conjugate-pair rule used blindly	first state coefficients are real
expanded high power	convert to polar and multiply angle

3 · Worked: elimination

C5 WORKED EXAMPLE

Q. Solve x+y+z=6, 2x−y+z=3, x+2y−z=2.

$R_2 \leftarrow R_2-2R_1$: (0,−3,−1 | −9)

$R_3 \leftarrow R_3-R_1$: (0,1,−2 | −4)

swap R_2, R_3

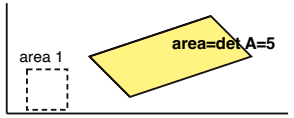
$R_3 \leftarrow R_3+3R_2$: (0,0,−7 | −21)

$z=3;\ y-2z=-4 \Rightarrow y=2;$

$x+y+z=6 \Rightarrow x=1$

check: 2(1)−2+3=3 ✓

Answer: (x,y,z)=(1,2,3), unique because every coefficient column is leading.



Determinant source-redraw: a linear map scales signed area by det A.

3b · Matrix traps & checks

C5–C6 FAQ

AB	inside dimensions match; entry=row·column; do not commute
A^{-1} by row reduction	$[A\|I]\to[I\|A^{-1}]$; zero row left means stop
parameter k	never divide by expression in k; reduce, then split zero/non-zero cases
homogeneous Ax=0	always consistent; non-leading column gives non-trivial solutions
inverse check	multiply back to I

For M=[[1,3,2],[2,7,5],[−1,2,4]]: R_2-2R_1, R_3+R_1 preserve det; det M=1·det[[1,1],[5,6]]=1 ⇒ invertible.

PARAMETER AND DETERMINANT DRILLS

For x+y+z=1, x+2y+3z=2, x+3y+kz=3, reduction gives the decisive row (0,0,k−5 | 0). If k≠5, every coefficient column pivots and the solution is unique. If k=5, the row is 0=0 and z is free, giving infinitely many solutions. There is no inconsistent case because the right entry remains zero.

GIVEN DET B=2 FOR 3×3 B	RESULT
det(B^T)	2
det(2B)	$2^3\cdot2=16$
det(B^{-1})	1/2
det(B^2)	4

Echelon read-off: a zero coefficient row with non-zero right side is a contradiction; a fully zero row is harmless. Do not count pivots in the augmented column as unknown pivots. Write free-variable solutions in vector form so the connection to a line or plane is visible.

4 · Worked: existence + uniqueness

COURSE WORKED EXAMPLE

Q. Show $x^3+2x-4=0$ has exactly one real root and locate it in an interval of length 0.1.

EXISTENCE

f is a polynomial ⇒ continuous.

f(1)=−1<0; f(2)=8>0.

IVT ⇒ a root in (1,2).

UNIQUENESS

$f'(x)=3x^2+2>0$ for all x

⇒ f strictly increasing ⇒ at most one root.

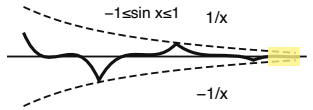
NARROW

f(1.1)=1.331+2.2−4=−0.469

f(1.2)=1.728+2.4−4=0.128

root ∈ (1.1,1.2).

Existence and uniqueness are separate claims.



Pinching source-redraw: both bounds tend to 0, forcing sin x/x to 0 as x→∞.

4b · Theorem trigger table

C7 + C9

WANTED	SAY FIRST	THEN
root exists	continuous on [a,b]	opposite signs + IVT
global extrema exist	continuous on closed [a,b]	MMT; evaluate candidates
equal chord/tangent slope	continuous [a,b], differentiable (a,b)	MVT equation
zero derivative between zeros	MVT hypotheses + equal endpoints	Rolle
inverse exists	strict monotonicity / one-to-one	swap domain/range

TRAP → *A sign change without continuity proves nothing. A continuous function on an open interval need not attain a maximum.*

MORE THEOREM DRILLS

Global extrema: for f(x)=x³−3x+1 on [−2,3], $f'=3x^2-3$, so candidates are −2,−1,1,3. Values are −1,3,−1,19; maximum 19 at 3, minimum −1 at both −2 and 1. Endpoints are mandatory.

Inverse derivative: $f(x)=x^5+2x+1$ has $f'=5x^4+2>0$, so it is one-to-one. Since f(1)=4, $(f^{-1})'(4)=1/f'(1)=1/7$. Evaluate f′ at the input mapping to 4, not at 4.

MVT number check: for f(x)=x²−4x on [1,4], chord slope=[f(4)−f(1)]/3=1. Solve 2c−4=1 to get c=5/2∈(1,4). State polynomial continuity and differentiability before the equation.

5 · Worked: improper integral

C11 WORKED EXAMPLE

Q. Determine $\int_0^\infty xe^{-x}dx$.

DEFINE IT

$\lim_{R\to\infty}\int_0^R xe^{-x}dx$

PARTS

$u=x,\ dv=e^{-x}dx \Rightarrow v=-e^{-x}$

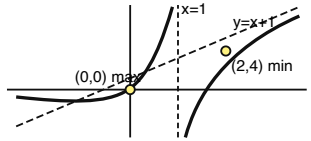
antiderivative = $-xe^{-x}-e^{-x}$

EVALUATE THE LIMIT

$[-xe^{-x}-e^{-x}]_0^R$

$=1-Re^{-R}-e^{-R} \to 1$

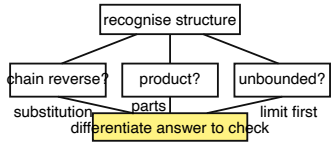
Both tail terms vanish because the exponential dominates every power; hence the integral converges to 1.



Curve-sketch source-redraw: $y=x^2/(x-1)$, with vertical x=1 and oblique y=x+1 asymptotes.

5b · Integration chooser

C11 FAQ



Method flow: structure selects substitution, parts or an improper-limit setup; differentiation verifies.

Definite substitution: change limits with the variable, then do not substitute back. **FTC chain:** upper limit g(x) contributes g′(x). **Signed area:** cancellation is allowed; geometric area needs absolute values or interval splitting.

INTEGRAL AND SKETCH DRILLS

$\int_0^1 2x(x^2+1)^3dx;\ u=x^2+1$

limits $1\to2 \Rightarrow \int_1^2 u^3du=[u^4/4]_1^2=15/4$

Changing limits removes the need to substitute back. Differentiate the antiderivative to check: $d[(x^2+1)^4/4]/dx=2x(x^2+1)^3$.

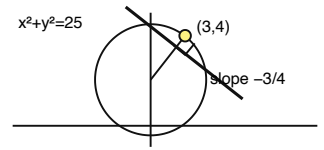
FTC chain drill: if $F(x)=\int_a^{g(x)}f(t)dt$, then $F'(x)=f(g(x))g'(x)$. The dummy variable t disappears; a non-constant lower limit would contribute its own negative term.

Curve sketch: for $y=(x^2+3)/(x-1)$, division gives $y=x+1+4/(x-1)$, so x=1 and y=x+1 are asymptotes. $y'=1-4/(x-1)^2$ vanishes at x=−1,3, giving (−1,−2) local maximum and (3,6) local minimum; $y''=8/(x-1)^3$ classifies them.

6 · Fast recall table

C8–C12

OBJECT	RULE / TRIGGER
inverse derivative	$(f^{-1})'(y)=1/f'(x)$, where f(x)=y
parametric slope	dy/dx=(dy/dt)/(dx/dt)
horizontal / vertical tangent	dy/dt=0 / dx/dt=0
oblique asymptote	polynomial division; remainder→0
polar → Cartesian	x=r cosθ, y=r sinθ, $r^2=x^2+y^2$
ln derivative	(ln x)′=1/x, x>0
a^x derivative	a^x ln a, a>0
variable power	ln y first
hyperbolic identity	$\cosh^2x-\sinh^2x=1$
p-integral	$\int_1^\infty x^{-p}$ converges iff p>1



Implicit-tangent source-redraw: y′=−x/y; tangent is perpendicular to the radius.

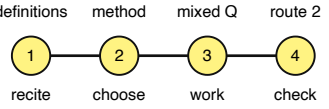
6b · Classic traps

ALL CHAPTER FAQS

Im(3+4i)	4, not 4i
cross product	b×a=−(a×b)
free variable	one parameter per non-leading coefficient column
det(cA)	c^ndet A, not c det A
differentiability	implies continuity; converse fails
L'Hôpital	only 0/0 or ∞/∞; not quotient rule
second derivative zero	no conclusion
improper integral	∞ is a limit, never a bound to substitute
cosh identity	minus sign: $\cosh^2-\sinh^2=1$

6c · Exam-morning plan

COURSE STUDY STRATEGY



Final pass: definitions and hypotheses first; then one mixed algebra and one mixed calculus solution.

Do one clean solution from memory, including theorem hypotheses and a concluding sentence. Mark every uncertainty; revisit only those rules. Check the exam time, room and current instructions on the LMS. Stop cramming new methods; rehearse selection and checking.

LAST-PAGE CHECK