# MAST20029 Engineering Mathematics

UNIVERSITY OF MELBOURNE · S2 2026 · CLOSED-BOOK MEMORY SHEET · FORMULA SHEET PROVIDED

Full guide + ask Sia about MAST20029

SIDE 1/2 DECIDE THE METHOD · CHECK CONDITIONS · INTERPRET THE OUTPUT

NOTHING OF YOUR OWN GOES IN



## 0 · Exam Map

COURSE DATA · BRIEF

Final 65% | MST 15% | 3×5% | Q

| | |
|---|---|
| **FINAL** | 65% · end of semester · in person · 15 min reading + 3 h writing · 11 questions · 120 marks |
| **OTHER** | Mid-semester test 15% · assignments (3 × 5%) 15% · quizzes (best 5 of 6) 5% |

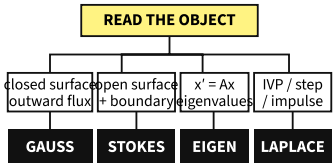
"You will be given the 5-page formula sheet below in the final exam:" *MAST20029 2026 SM1 Canvas exam-information page; 2026 SM1 exam front cover; 2025 S1 paper – confirm this semester's rule on the LMS.*

**What the exam sheet gives you:** the named *5-page formula sheet*. This memory sheet instead drills the choices it cannot make: method, conditions, orientation, limits, interpretation and justified working.

**EXAM HABIT →** *Final marks reward correct method, full working, accuracy and notation. State the theorem/test; check the condition that licenses it; then execute.*

## 1 · Trigger → Method

ALL SECTIONS




**series: shape → test** **PDE: Δ=B²−4AC**

**NAME · CONDITION · WORKING · CONCLUSION**

Master selector: geometry or algebra supplies the trigger; the named condition supplies the mark.

| WORDING | FIRST MOVE |
|---|---|
| Impossible inner antiderivative | Sketch region; change order |
| Disk / cylinder / ball | Match coordinates + Jacobian |
| Closed-loop work | Test curl F=0 first |
| Factorial / nth power | Ratio / root test |
| Periodic symmetry | Odd→sine; even→cosine |
| Rectangle + homogeneous edges | Separate; BCs set modes |

**ELEVEN-QUESTION RETRIEVAL ROUTE**

| FAMILY | RECALL BEFORE ALGEBRA |
|---|---|
| Vector calculus | object, geometry, orientation, Jacobian |
| ODE systems | eigenvalue case; phase type; stability |
| Laplace | initial terms; shift/impulse/convolution signal |
| Series | term limit first; test from shape; endpoints |
| Fourier | period L; parity; value at jumps |
| PDE | class; homogeneous direction; match edge last |

## 2 · Vector Operators

CH 1 · Q1–Q3

| INPUT → OUTPUT | MEANING / CHECK |
|---|---|
| ∇ϕ → vector | steepest increase; normal to level sets |
| ∇·F → scalar | net outflow: + source, − sink |
| ∇×F → vector | local rotation; determinant signs + − + |

**INSTANT IDENTITIES**

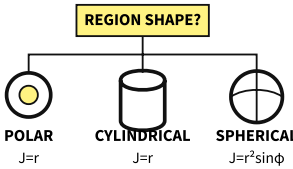
```
curl(grad φ)=0  ·  div(curl F)=0
```

Conservative test: on a **simply connected** domain, curl F=0 ⇔ F=∇ϕ. Recover ϕ by integrating one component; the "constant" is a function of the other variables, fixed by matching.

**Exam bite:** keep scalar divergence and three-component curl visibly distinct; the *j*-component carries the negative cofactor. Use the identities both as explanations and as arithmetic checks.

## 3 · Region → Coordinates

CH 2–3




**limits first → substitute → multiply Jacobian**

Symmetry chooser. Course convention: ϕ is measured down from +z; θ is azimuth.

| SYSTEM | SUBSTITUTION | BEST FOR |
|---|---|---|
| Polar | x=r cosθ, y=r sinθ | disks, annuli, $x^2+y^2$ |
| Cylindrical | polar + z unchanged | cylinders / axis symmetry |
| Spherical | x=r cosθ sinϕ; y=r sinθ sinϕ; z=r cosϕ | balls, spheres |

| GEOMETRIC PHRASE | TYPICAL LIMIT |
|---|---|
| first quadrant | 0≤θ≤π/2 |
| full disk/cylinder | 0≤θ≤2π |
| full ball | 0≤θ≤2π, 0≤ϕ≤π |
| radius a | 0≤r≤a |

**Change-order routine:** draw R; new outer variable takes its full numeric range; new inner limits are the curves met by a strip. Never exchange symbols mechanically. An inner ∫e^(x³)dx is the classic swap signal.

**TWO SETUP DRILLS**

**Quarter-annulus:** x,y≥0 and $1\le x^2+y^2\le 9$ gives 1≤r≤3, 0≤θ≤π/2. With density $1/(x^2+y^2)$, density × Jacobian =1/r, so mass=(π/2)ln3. The answer diagnoses the setup: without J=r the radial integral would be wrong.

| | |
|---|---|
| **BEFORE INTEGRATING** | sketch → system/order → bounds → Jacobian |
| **AFTER INTEGRATING** | range/sign/units plausible? region covered once? |

## 4 · Curves & Surfaces

CH 4

| ASKED FOR | SET-UP |
|---|---|
| Scalar line | $\int f(r(t))\lvert r'(t)\rvert dt$ |
| Work | $\int F(r(t))\cdot r'(t)dt$ |
| Surface scalar | $\iint g\sqrt{(f_x^2+f_y^2+1)}dA$ |
| Upward flux | $\iint(-F_1f_x-F_2f_y+F_3)dA$ |

**Before work:** compute curl F. If zero on a simply connected domain, find ϕ and use $\int_C F\cdot dr=\phi(\text{end})-\phi(\text{start})$. A closed loop then gives 0 without edge-by-edge integration.

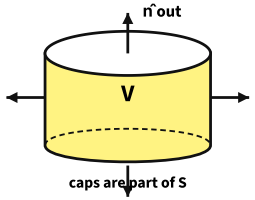
**Orientation:** increasing t directs r(t); reversing the path reverses work. "Downward" reverses an upward-flux sign.

| OBJECT | PARAMETRISE / PROJECT | CHECK |
|---|---|---|
| circle radius a | r(t)=(a cos t,a sin t,0) | t increasing is anticlockwise from +z |
| segment A→B | r(t)=tB+(1−t)A, 0≤t≤1 | r(0)=A, r(1)=B |
| graph z=f(x,y) | project to R in xy-plane | use the correct scalar or flux factor |

**Do not merge the two surface formulas.** The square-root factor belongs to scalar accumulation/area; the vector normal factor belongs to flux. Ask whether the integrand is a scalar g or a field F before choosing.

## 5 · Gauss

CH 5 · FLUX TRIGGER




Source redraw, Ch 5: Gauss needs the whole **closed** boundary and outward normal.

**LICENCE + CONVERSION**

```
closed S=∂V, outward n̂ ⇒ ∬S F·n̂ dS = ∭V ∇·F dV
```

Use Gauss when divergence over the solid is easier than summing face fluxes. For an open side, add the cap, apply Gauss, then subtract cap flux. Choose coordinates from V and carry the Jacobian.

**CHECK →** *constant divergence collapses to constant × volume.*

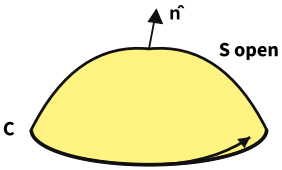
**CLOSED-SURFACE MICRO-DRILL**

For F=(x,y,z) on the sphere $x^2+y^2+z^2=a^2$, ∇·F=3. Gauss gives flux=$\iiint_V 3dV=3\cdot(4/3)\pi a^3=4\pi a^3$. This is not "formula recall": the marks lie in recognising **closed + outward** and converting to constant × volume.

| GIVEN SURFACE | LEGAL ROUTE |
|---|---|
| whole sphere / closed cylinder | Gauss immediately |

## 6 · Stokes

CH 6 · CIRCULATION




**right-hand rule: fingers C, thumb n̂**

Source redraw, Ch 6: boundary direction and normal are one compatible orientation.

**LICENCE + CONVERSION**

```
open oriented S with boundary C ⇒ ∬S (∇×F)·n̂ dS = ∮C F·dr
```

**Choose the easier side.** Simple curl → surface flux; simple boundary → circulation. Replace an awkward cap by any easier surface sharing C. Reversing C or n̂ flips the sign.

| FEATURE | GAUSS | STOKES |
|---|---|---|
| Surface | closed | open |
| Operator | div | curl |
| Other side | volume | boundary |
| Orientation | outward | right-hand |

**CHOICE TEST**

| GIVEN / REQUESTED | COMPARE |
|---|---|
| circulation around simple circle | direct parametrisation versus curl over flat disk |
| curl flux through awkward cap | same boundary, replace by flat surface |
| conservative F around loop | quote zero; no integral needed |
| surface without boundary | not Stokes; inspect Gauss if closed |

**Sign sentence:** "The upward normal pairs with anticlockwise traversal as viewed from above." If the problem states the opposite traversal, either reverse C or keep n̂ and attach a minus sign—never silently mix conventions.

## 7 · Guardrails

SHORT-ANSWER MARKS

- curl F=0 needs a simply connected domain.
- Gauss needs closure; Stokes needs C=∂S.
- Keep the same boundary when changing a Stokes surface.
- The Jacobian belongs to dA/dV after coordinate change.
- Name the theorem before substitution.

```
curl(grad φ)=0 · div(curl F)=0
```

## 7b · Stokes Micro-Drill

CH 6 WORKED

| | |
|---|---|
| **SURFACE INDEPENDENCE** | same C + compatible orientation → same curl flux |
| **SIGN CHECK** | viewed from n̂, positive C is anticlockwise |

## 8 · Systems x′=Ax

CH 7 · Q4

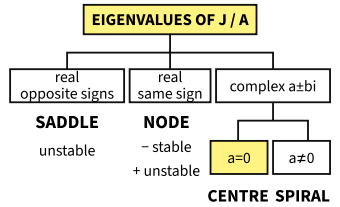
Pipeline: det(A−λI)=0 → eigenvalues → eigenvectors → correct solution form → initial conditions. Eigenvector directions are straight-line orbits; sign(Re λ) controls in/out behaviour.

| EIGENVALUES | SOLUTION MOVE |
|---|---|
| distinct real | $\alpha_1 w_1 e^{(\lambda_1 t)}+\alpha_2 w_2 e^{(\lambda_2 t)}$ |
| repeated, two w | two independent eigenmodes |
| deficient | (wt+u)e^(λt), (A−λI)u=w |
| a±bi | Re/Im of we^(λt); no i remains |

A centre requires pure imaginary eigenvalues: real part *exactly* zero. Asymptotic stability instead needs negative real parts.

## 9 · Classify & Sketch

CH 8 · Q5




**repeated λ: check geom. mult.**

Classification is immediate; the marks move to eigen-lines, arrows, slopes and long-time behaviour.

1. Name type + stability.
2. Draw eigenvector lines; λ>0 out, λ<0 in.
3. Add trajectories and t→±∞ asymptotes.
4. Use dy/dx=ẏ/ẋ for axis slopes.
5. Non-linear: solve critical points, evaluate J at each, classify locally. Borderline centres may be inconclusive.

| SPECTRUM | TYPE | FLOW |
|---|---|---|
| $\lambda_1,\lambda_2$<0 real | stable node | both modes in |
| $\lambda_1,\lambda_2$>0 real | unstable node | both modes out |
| $\lambda_1\lambda_2$<0 | saddle | one in, one out |
| a±bi, a<0 | stable spiral | spirals in |
| a±bi, a>0 | unstable spiral | spirals out |
| ±bi | centre | closed orbits |

**Linearisation frame:** solve f(x,y)=g(x,y)=0; form J=[[$f_x$,$f_y$],[$g_x$,$g_y$]]; substitute each critical point; classify its eigenvalues separately. For spirals/centres, determine rotation from one test velocity vector rather than guessing.

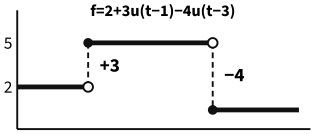
**Eigenvector geometry:** along w, Aw=λw remains on the same line, so the orbit can be straight. A generic initial vector combines modes and bends between eigen-directions. For a saddle, the negative-λ line is the stable direction and the positive-λ line the unstable direction.

## 10 · Laplace Recognition

CH 9 · Q6

Pipeline: transform IVP with initial terms → solve X(s) → partial fractions → invert → **name** each shift/convolution theorem.

| SIGNAL | DECISION |
|---|---|
| s replaced by s+a | s-shift |
| e^(−as) | t-shift; delayed u(t−a) |
| δ(t−a) | transform e^(−as) |
| $\int_0^t f(\tau)g(t-\tau)d\tau$ | convolution → F(s)G(s) |
| piecewise forcing | jumps × unit steps |




Source redraw, Ch 9: each step coefficient is the **jump**, not the new height.

**DERIVATIVES**

```
L{x'}=sX-x(0)
L{x''}=s²X-sx(0)-x'(0)
```

## 11 · Series & Fourier

CH 10–11

| SHAPE | CHOOSE |
|---|---|
| $a_n\to 0$ | divergence: diverges |
| bounded / $n^p$ | comparison with p-series |
| n!, $c^n$ | ratio; L=1 silent |
| $(\dots)^n$ | root; L=1 silent |
| $(-1)^n b_n$ | Leibniz, then absolute |

**Power series:** find R, then test both endpoints. **Fourier:** even→cosine; odd→sine. At a jump use ½[f(t⁻)+f(t⁺)].

**BENCHMARKS**

```
Σarⁿ converges iff |r|<1; sum=a/(1-r)
Σ1/nᵖ converges iff p>1
```

**Taylor move:** identify the standard series, match the centre, then substitute the requested value; for approximation, the polynomial matches f and its derivatives at the centre and the remainder controls the gap.

## 12 · PDE Decision

CH 12 · Q11

| ELLIPTIC | PARABOLIC | HYPERBOLIC |
|---|---|---|
| $B^2-4AC<0$ | =0 | >0 |
| **Laplace** | **diffusion** | **wave** |
| steady state | heat spreads | travels at c |

**classify → separate → modes → match**

Source redraw, Ch 12: the discriminant names the physics; boundary conditions select modes.

| SEPARATION STAGE | WHAT TO SHOW |
|---|---|
| Product | u=XY (or XT for time) |

# MAST20029 Engineering Mathematics · Self-test Drills

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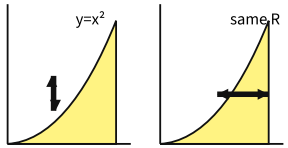
SIDE 2/2 WHICH-IS-TRUE TRAPS · SHORT-ANSWER FRAMES · WORKED APPLICATIONS

REVISE · RETRIEVE · JUSTIFY

Memory sheet for revision · Side 2

## 13 · Integral Judgement CH 2–6

| WHICH IS TRUE? | CORRECT CONTRAST |
|---|---|
| Swap by exchanging dx,dy? | **No:** sketch R; re-read strips. |
| Jacobian optional? | **No:** it scales dA/dV. |
| curl F=0 always conservative? | Only on simply connected domain. |
| Gauss on curved wall alone? | Close with caps, then correct. |
| Stokes surface fixed? | No; boundary fixed, surface may change. |




Source redraw, Ch 2: changing order re-describes the same region; outer limits are numeric extremes.

## 14 · Worked A · Change Order 5 MARKS

**Evaluate** $\int_0^1\int_y^1 e^{(x^3)}\,dx\,dy$.

1. $0\le y\le 1$, $\sqrt{y}\le x\le 1 \Rightarrow y\le x\le x^2$.
2. Same R: $0\le x\le 1$, $0\le y\le x^2$.
3. Swap: $\int_0^1\int_0^{x^2} e^{(x^3)}dy\,dx = \int_0^1 e^{(x^3)}x^2dx$.
4. $u=x^3$, $du=3x^2dx \Rightarrow$ (e−1)/3.

The picture licenses the swap and exposes the substitution.

## 15 · Worked B · Gauss 8 MARKS

$F=(x^3,y^3,z^3)$; closed cylinder $x^2+y^2\le 4$, $0\le z\le 2$.

```
∇·F=3(x²+y²+z²)
dV=r dr dθ dz
```

Flux$=3\int_0^{2\pi}\int_0^2(r^2+z^2)r\,dz\,dr\,d\theta$. Inner z: $2r^2+8/3$; then $\int_0^{2\pi}(2r^3+(8/3)r)dr=40/3$. Restore 3 and $2\pi$: 80π.

**TRAP →** *dropping the Jacobian r changes the problem.*

## 15b · Conservative Drill CH 4 WORKED

$F=(2xy+z^2, x^2, 2xz)$. Mixed partials match, so curl F=0 on the stated simply connected domain. Integrate $\phi_x=2xy+z^2$:

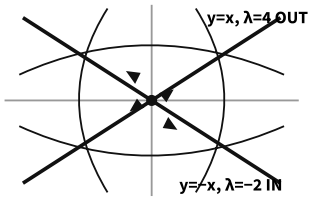
| | |
|---|---|
| **DIRECT PATH** | needed when curl F≠0 or domain condition fails |
| **POTENTIAL SHORTCUT** | endpoint difference; closed loop gives zero |

## 15c · Polar Drill CH 3 WORKED

| | |
|---|---|
| **REGION CUE** | annulus + first quadrant |
| **ALGEBRA CUE** | $x^2+y^2$ becomes $r^2$ |
| **ESSENTIAL FACTOR** | J=r turns $1/r^2$ into $1/r$ |

## 16 · ODE Which-Is-True CH 7–8

| CLAIM | VERDICT / REPAIR |
|---|---|
| Opposite-sign λ is stable node. | **False:** saddle; unstable. |
| Pure imaginary λ gives spiral. | **False:** centre. |
| Repeated λ always has two w. | **False:** check geometric multiplicity. |
| λ<0 arrows point out. | **False:** e^(λt) decays; point in. |
| A sketch alone is enough. | **False:** justify lines, slopes, limits. |




Source redraw, Ch 8 worked saddle: λ=4 along y=x; λ=−2 along y=−x.

## 17 · Worked C · Saddle 7 MARKS

x′=x+3y, y′=3x+y.

1. λ=1±3 ⇒ 4,−2 ⇒ unstable saddle.
2. $w_1=(1,1)$: y=x, outgoing. $w_2=(1,-1)$: y=−x, incoming.
3. t→+∞: generic orbits follow y=x; t→−∞: y=−x.
4. dy/dx=(3x+y)/(x+3y): y-axis slope 1/3; x-axis slope 3.

**Frame:** "Eigenvalues __ hence __; eigen-lines __ with arrows __; slopes __; therefore t→±∞ __."

## 18 · Centre Drill CH 7 WORKED

x′=ax−by, y′=x+ay gives λ=a±√(−b). Centre: a=0,b>0. If a=0,b=4, λ=±2i and w=(2,−i):

```
x(t)=a₁(2cos2t,sin2t)+a₂(2sin2t,
-cos2t)
```

No i remains; closed orbits are neutral, not asymptotically decaying.

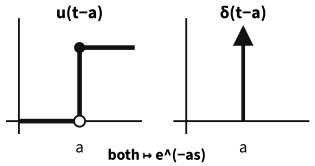
## 18b · Sketch Checklist JUSTIFY EVERY FEATURE

| FEATURE | EVIDENCE TO WRITE |
|---|---|
| type | eigenvalue signs / real parts |
| straight orbits | eigenvector lines |
| arrow direction | growth/decay of e^(λt) |
| axis crossings | dy/dx=y/x at x=0 or y=0 |
| rotation | evaluate one velocity vector |
| non-linear point | Jacobian evaluated at that point |

**Repeated-eigenvalue trap:** two independent eigenvectors give the ordinary two-mode form; a deficient matrix has one eigenvector and needs a generalised eigenvector u satisfying $(A-\lambda I)u=w$, producing the t e^(λt) behaviour.

## 19 · Laplace Judgement CH 9

| WHICH IS TRUE? | ANSWER |
|---|---|
| e^(−2s) changes s to s+2. | **False:** delay in t. |
| δ(t−a)→e^(−as). | **True.** |
| Step coefficient is new level. | **False:** it is the jump. |
| Convolution becomes a sum. | **False:** product F(s)G(s). |
| Initial terms can be added later. | **False:** include immediately. |




Source redraw, Ch 9: a delayed step persists; a Dirac delta is an idealised unit impulse.

## 20 · Worked D · Impulse IVP 6 MARKS

x″+3x′=δ(t−2), x(0)=1, x′(0)=0.

1. $(s^2X-s)+3(sX-1)=e^{(-2s)}$.
2. $X=1/s+e^{(-2s)}/[s(s+3)]$.
3. $1/[s(s+3)]=(1/3)[1/s-1/(s+3)]$.
4. By **t-shifting**, delayed inverse is (1/3)[1−e^(−3(t−2))]u(t−2).

**ANSWER**

```
x(t)=1+(1/3)[1-e^(-3(t-2))]u(t-2)
```

Before t=2, u=0 and x=1; the exponential contains (t−2).

## 21 · Answer Frames METHOD MARKS

**Shift:** "X=e^(−as)F; by t-shifting, x=f(t−a)u(t−a)."

**Convolution:** "Integral=(y∗g)(t), hence transform YG; solve for Y."

**Piecewise:** "Initial level + Σ(new−old)u(t−a)."

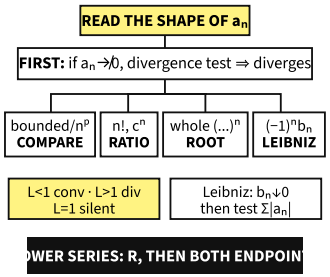
## 21b · Transform Audit BEFORE INVERSION

| AUDIT QUESTION | FAILURE IT CATCHES |
|---|---|
| Did every derivative include initial data? | lost constants in X(s) |
| Is the rational part proper? | partial fractions before division |
| What base pair is on the table? | wrong sine/cosine/exponential scale |
| Is e^(−as) factored outside? | delay applied to only part of f |
| Did I name the theorem? | missing explicit method mark |

**STEP-FUNCTION RECONSTRUCTION**

| | |
|---|---|
| **S-SHIFT** | multiplication by e^(−at) shifts F(s) to F(s+a) |
| **T-SHIFT** | factor e^(−as) delays the whole base inverse and adds u(t−a) |

## 22 · Convergence Selector CH 10 · Q7–8




Source redraw, Ch 10: the divergence test comes first; a zero term limit proves nothing.

| TRAP | REPAIR |
|---|---|
| L=1 | choose another test |
| alternating converges | may be conditional; test absolute |
| radius found | still test endpoints |
| sin n numerator | bound it; do not evaluate |

## 23 · Application · Comparison + R 6 MARKS

**(a)** $\Sigma(2+\sin n)/(n^2+1)$:

```
0≤(2+sin n)/(n²+1)≤3/n²
```

$\Sigma 3/n^2$ converges (p=2>1), so the original converges by comparison.

**(b)** $\Sigma n!(x-3)^n/n^n$:

```
L=(n/(n+1))ⁿ|x-3| → |x-3|/e
```

L<1 ⇒ |x−3|<e, so R=e. If the interval is requested, test both endpoints.

**SAY IT →** *Name the test and quote the inequality/limit that licenses the conclusion.*

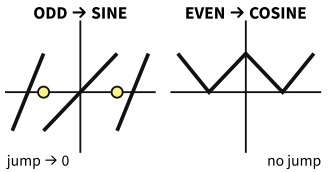
## 23b · Fast Recall Table CONDITIONS MATTER

| TEST | REQUIRED SHAPE | CONCLUSION |
|---|---|---|
| comparison | non-negative terms; useful upper/lower benchmark | direction must support claim |
| limit comparison | $a_n,b_n>0$ and $a_n/b_n\to c\in(0,\infty)$ | same convergence fate |
| integral | positive, continuous, decreasing | Σ and improper integral agree |
| Leibniz | $b_n$ decreases to 0 | alternating series converges |
| ratio/root | absolute-value limit exists | L<1 conv; L>1 div |

**TAYLOR RECALL**

| | |
|---|---|
| **APPROXIMATION QUESTION** | write polynomial + remainder/error statement |
| **NUMERICAL-SUM QUESTION** | name standard series + substitution value |
| **POWER SERIES** | radius first; interval needs endpoints |

## 24 · Fourier Judgement CH 11 · Q9–10




Source redraw, Ch 11: reflect through the origin for odd; across the vertical axis for even; repeat with period 2L.

| WHICH IS TRUE? | CORRECT RULE |
|---|---|
| Odd extension has cosine. | **False:** $b_n$ sine only. |
| Even extension has sine. | **False:** $a_0,a_n$ cosine only. |
| At a jump series equals f. | **False:** one-sided midpoint. |
| Non-periodic uses series. | Use Fourier integral. |

Decide symmetry first. On (0,L), use factor 2/L; keep ω=π/L consistent; show integration-by-parts boundary terms.

## 25 · Application · Sine Series 6 MARKS

f(t)=t, 0<t<2, odd extension: L=2, ω=π/2.

```
bₙ=∫₀²t sin(nπt/2)dt
=4(-1)^(n+1)/(nπ)
```

**SERIES**

```
f(t)=Σ_{n≥1}[4(-1)^(n+1)/(nπ)]sin(nπt/2)
```

At t=2 the extension jumps; the series tends to 0, the midpoint.

## 26 · Answer Frames CONVERGENCE

**Symmetry:** "The extension is odd/even, so __ coefficients vanish."

**Jump:** "By midpoint convergence, limit=½[f(t⁻)+f(t⁺)]=__."

**Integral:** "Non-periodic; use continuous frequency, cosine-only if even or sine-only if odd."

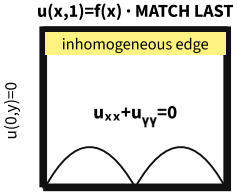
## 26b · Fourier Fast Recall CH 11

| PROMPT | JUDGEMENT BEFORE INTEGRATION |
|---|---|
| f given on (0,L) | choose odd/even periodic extension |
| odd extension | $a_0=a_n=0$; compute $b_n$ over (0,L) |
| even extension | $b_n=0$; compute $a_0,a_n$ over (0,L) |
| jump point | report midpoint of one-sided limits |
| periodic forcing | expand forcing; solve harmonics term-by-term |
| non-periodic signal | Fourier cosine/sine integral by parity |

**Gibbs distinction:** partial sums develop persistent overshoot near a jump, while at the jump itself the series converges exactly to the midpoint. Do not confuse local overshoot with the pointwise limit.

**COEFFICIENT AUDIT**

## 27 · PDE Rectangle CH 12 · Q11




Source redraw, Ch 12: three homogeneous edges select modes; the remaining edge fixes coefficients.

| STAGE | MARK-BEARING LINE |
|---|---|
| Separate | u=XY ⇒ X″/X=−Y″/Y=−λ |
| BC problem | X(0)=X(1)=0 ⇒ $\lambda_n=(n\pi)^2$, $X_n=\sin(n\pi x)$ |
| Other factor | $Y_n=\sinh(n\pi y)$, vanishes at y=0 |
| Superpose | $u=\Sigma c_n\sin(n\pi x)\sinh(n\pi y)$ |
| Match | impose u(x,1)=f(x) last |

## 28 · Application · Laplace Square 8 MARKS

If u(x,1)=sin(2πx)+3sin(5πx), matching gives $c_2\sinh(2\pi)=1$, $c_8\sinh(5\pi)=3$; all other $c_n=0$.

**SOLUTION**

```
u=[sin(2πx)sinh(2πy)]/sinh(2π)
+3[sin(5πx)sinh(5πy)]/sinh(5π)
```

Do not integrate coefficients when data is already a finite sine sum. Check all four boundary conditions.

## 28b · PDE Trap Table LONG-QUESTION DEFENCE

| TRAP | CORRECTION |
|---|---|
| classify by summing coefficients | use $B^2-4AC$ |
| write only one λ case | show why other signs are trivial |
| choose cosh for Y(0)=0 | choose sinh, which vanishes |
| fit f(x) too early | homogeneous BCs select modes first |
| integrate a finite sine sum | match coefficients directly |
| omit final check | substitute every boundary edge |

## 29 · Exam-Morning Plan REVISION ONLY

TRIGGERS CONDITIONS ONE DRILL STOP

1. method selector
2. theorems tests + BCs
3. weakest section
4. exact forms calm setup

Final retrieval loop: choices and conditions, not a fresh topic.