MAST10007 Linear Algebra

UNIVERSITY OF MELBOURNE · FIRST-YEAR UNDERGRADUATE · SEMESTER 2, 2026

Full guide + ask Sia about MAST10007

SIDE 1/2 METHODS · conditions → steps → checks ONE A4 NOTE SHEET Final 60% · three hours · **method map**

0 · Exam map

COURSE FACTS

Final 60% | Mid | MATLAB | 4

ASSESSMENT	WEIGHT · FORMAT
Final written examination	**60%** · 3 h · end-of-semester period
Online mid-semester test	10% · 45 min · Week 6 · open book
MATLAB test	15% · 45 min · Week 12 · restricted materials
Four written assignments	15% total · equally weighted

"The only permitted material for the final exam is one A4 sheet of notes, with writing back and front."
(MAST10007 subject materials, 2026)

1 · Row reduction

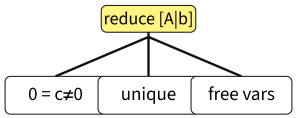
CH 1–2 · SOLVE/CLASSIFY

★ Fix variable order; encode **all** coefficients and constants in [A|b]. Legal reversible moves: $R_i \leftrightarrow R_j$; $R_i \leftarrow cR_i$ with $c\neq 0$; $R_i \leftarrow R_i + cR_j$. Apply each move across the augmented bar.

EXAM ROUTINE

```
leftmost useful pivot → swap if zero
→ clear below → continue → back-
substitute; use exact arithmetic →
substitute into original equations.
```

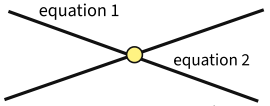
REDUCED EVIDENCE	CONCLUSION
[0 … 0 | c], c≠0	inconsistent · no solution
consistent; pivot every variable	unique solution
consistent; ≥1 free variable	infinitely many; one parameter per free variable
[0 … 0 | 0]	redundant row only



Classify only after checking the augmented column, then the free-variable count.

1b · What the rows mean

CH 1–2 · INTERPRET



Row operations change the equations but preserve their common solution set.

2 · Matrix algebra

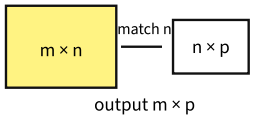
CH 3 · STRUCTURE FIRST

PRODUCT

```
A_{m×n}B_{n×p} exists → AB is m×p; (AB)_ij =
row_i(A)·col_j(B). Usually AB≠BA; one
order may not exist.
```

TRANSPOSE / INVERSE

```
(AB)^T=B^TA^T · (AB)^{-1}=B^{-1}A^{-1} · solve
AX=I or reduce [A|I]. Never "divide
by A".
```



Product gate: inner sizes must match; outer sizes predict the answer.

2×2 DETERMINANT AND INVERSE

```
A=[[a,b],[c,d]] · det A=ad-bc.
det A≠0 ⇒ A^{-1}=(1/(ad-bc))[[d,-b],
[-c,a]].
```

FOR SQUARE N×N A, EQUIVALENT
invertible ⇔ det A≠0 ⇔ rank A=n ⇔ pivot every row/column ⇔ Ax=b unique for every b ⇔ ker A={0}

Determinant row effects: swap rows → sign flips; scale a row by c → determinant scales by c; row replacement → determinant unchanged. For triangular A, det A is the product of diagonal entries.

3 · Vector geometry

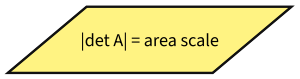
CH 4 · CHOOSE OUTPUT

```
u·v=Σ_i u_i v_i = ||u|| ||v|| cosθ · u⊥v ⇔
u·v=0
proj_u(v)=((v·u)/(u·u))u, u≠0
r=v-proj_u(v) ⇒ r·u=0.
```

QUESTION ASKS	USE
angle, orthogonality, projection	dot product
normal to two R^3 directions; area	u×v; ‖u×v‖
line	x=p+td, d≠0
plane	n·(x−p)=0

3b · Determinant geometry

CH 3–4 · SCALE

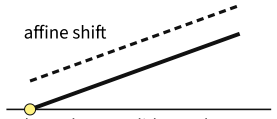


det A=0 means dimension collapses, so an inverse cannot exist.

4 · Subspace or affine?

CH 5 · PROOF

Subspace test: for W⊆V, show 0∈W and for arbitrary u,v∈W, scalars a,b, au+bv∈W. A homogeneous linear constraint has a subspace solution set; a translated constraint may be affine.



Passing through zero is necessary, not sufficient: closure completes the proof.

FAST COUNTEREXAMPLE	WHAT IT DISPROVES
0∉W	not a subspace immediately
u,v∈W but u+v∉W	addition closure fails
u∈W but cu∉W	scalar closure fails

5 · Span · independence

CH 6–7 · PIVOT LOGIC

TWO DIFFERENT SYSTEMS

```
b∈span{v_1,…,v_k}? Solve [v_1 … v_k]c=b;
consistency gives coefficients.
Independent? Solve [v_1 … v_k]c=0;
only c=0.
```

A nontrivial relation identifies redundancy: isolate any vector with nonzero coefficient; removing it preserves the span. The zero vector makes a list dependent. More vectors than ambient dimension forces dependence.

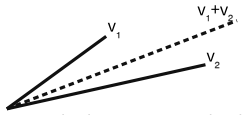
Basis = independent + spanning; coordinates are unique only relative to a declared **ordered** basis B. If $v=\Sigma c_i b_i$, then $[v]_B=(c_1,\ldots,c_n)$.

FUNDAMENTAL-SPACE BOOKKEEPING

WANTED	TAKE FROM	DIMENSION
Col(A) basis	original pivot columns	rank r
Row(A) basis	nonzero echelon rows	r
Nul(A) basis	special solutions Ax=0	n−r
Nul(A^T)	solve $A^Ty=0$	m−r

5b · Redundancy picture

CH 6 · EXPLAIN RELATION



A nontrivial dependence relation tells you exactly which generator can be removed.

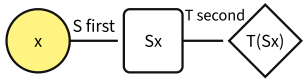
6 · Linear maps

CH 8–9 · BUILD/DIAGNOSE

LINEARITY

```
T(u+v)=T(u)+T(v) and T(cu)=cT(u).
Hence T(0)=0 and
T(au+bv)=aT(u)+bT(v).
```

For $T:R^n \to R^m$, the standard matrix is $A=[T(e_1)\ \ldots\ T(e_n)]$ of size m×n. A nonzero translation fails T(0)=0. If S acts first, then T, the composite matrix is A_TA_S.



Read products right-to-left: first S, then T gives A_TA_S.

PROPERTY	EQUIVALENT TEST
one-to-one	ker T={0}; pivot every column
onto codomain R^m	im T=R^m; pivot every row
invertible	one-to-one and onto

Kernel = solutions of Ax=0, living in the domain. *Image* = Col(A), living in the codomain. A nonzero kernel vector records a lost input direction.

7 · Change of basis

CH 7, 9 · DIRECTION

If $P^B=[b_1\ \ldots\ b_n]$, then $v=P^B[v]_B$. Thus $[v]^C=(P^C)^{-1}P^B[v]_B$. Label source and destination; the reverse conversion is the inverse.

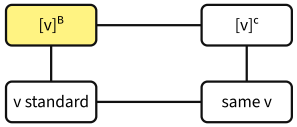
SAFEST CHECK

```
source coordinates → reconstruct
standard v → solve in destination
basis → reconstruct the same
standard v.
```

DIRECTION ERROR	REPAIR
used basis columns on coordinate vector	that maps basis coordinates → standard
need standard → basis	apply inverse basis matrix

7b · Coordinate diagram

CH 9 · COMMUTE



If the two routes disagree, the conversion direction or basis order is wrong.

COMPOSITION AUDIT → *Send a basis vector through the maps in order; it must match the corresponding column of the product. The first map is rightmost. Domain and codomain belong to T.*

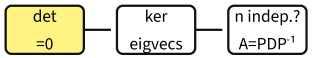
8 · Eigenstructure

CH 10 · DIAGONALISE

METHOD

```
det(A-λI)=0 → for each λ solve
(A-λI)v=0 → collect independent
eigenvectors → P=[v_1 … v_n],
D=diag(λ_1,…,λ_n) in matching order.
```

An eigenvector is nonzero and obeys Av=λv. Algebraic multiplicity counts roots; geometric multiplicity is dim eigenspace. Always 1≤geom≤alg.



Diagonalisation is licensed by n independent eigenvectors, not merely n roots counted with repetition.

```
A diagonalizable ⇔ n independent
eigenvectors.
Distinct eigenvalues ⇒ independent
eigenvectors.
A=PDP^{-1} ⇒ A^k=PD^kP^{-1}.
```

CHECK → *Verify AP=PD before computing P^{-1}. Repeated λ needs an eigenspace-dimension check.*

9 · Powers · Markov

CH 11 · CONVENTION

Recursion $x_{k+1}=Ax_k$ gives $x_k=A^kx_0$. For column states $x_{k+1}=Px_k$, **columns** sum to 1; row-state convention uses row sums.

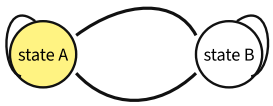
STATIONARY STATE

```
Px=x, x_i≥0, Σx_i=1. Solve eigenvalue
1, then normalise.
```

CLAIM	REQUIRED CHECK
valid probability state	nonnegative; sums to 1
stationary	one update returns same state
long-run convergence	other eigenstructure/modes decay

9b · State motion

CH 11 · INTERPRET

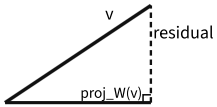


Declare row/column convention before reading transitions or checking sums.

10 · Orthogonality

CH 12 · PROJECT/GS

```
||v|| = √(v,v) · u⊥v ⇔ (u,v)=0
proj_u(v)=((v,u)/(u,u))u, u≠0.
Orthonormal {q_i}: proj_W(v)=Σ(v,q_i)q_i.
```



A correct projection lies in W and leaves a residual orthogonal to every basis direction.

Gram–Schmidt: $u_i=v_i$; $u_j=v_j-\Sigma_{i<j}\,\mathrm{proj}_{u_i}(v_j)$; then $q_j=u_j/\|u_j\|$. Subtract every earlier projection; the resulting nonzero list preserves the original span.

BASIS TYPE	PROJECTION TERM
orthogonal u_i	$\langle v,u_i\rangle/\langle u_i,u_i\rangle u_i$
orthonormal q_i	$\langle v,q_i\rangle q_i$

11 · Least squares · SVD

CH 13–14 · FIT/FACTOR

LEAST SQUARES

```
x̂ minimises ||b-Ax||; r=b-Ax̂ ⊥ Col(A).
A^TA x̂=A^Tb. Unique if columns of A are
independent.
```

Orthogonal Q: $Q^TQ=I$, $Q^{-1}=Q^T$; preserves norms, dot products and angles. Real symmetric A has real eigenvalues and an orthonormal eigenbasis: $A=QDQ^T$.

SVD

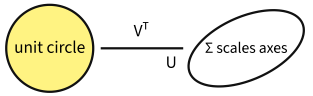
```
A=UΣV^T: V^T selects orthonormal input
directions → Σ scales by nonnegative
singular values → U places orthonormal
output directions.
```

Complex: $A^*=\bar{A}^T$; $(AB)^*=B^*A^*$. Hermitian $H^*=H$. Unitary $U^*U=I$, so $U^{-1}=U^*$. For real entries these become symmetric and orthogonal.

TRAP → *Normal equations express closest fit, not exact solvability; singular values are nonnegative and are not generally eigenvalues of A.*

11b · SVD geometry

CH 13 · ALL REAL A



Right singular vectors select input axes; singular values stretch; U orients outputs.

Your exam allows one A4 sheet of notes, both sides. Check on the LMS whether printed notes are accepted; if they must be handwritten, copy it.

Forward this to your tutorial group

SHARED BY _____

MAST10007 Linear Algebra · worked drills and recall

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Full guide + ask Sia about MAST10007

SIDE 2/2 FIVE WORKED DRILLS · traps · fast recall — WRITE → CHECK → INTERPRET — Keep exact arithmetic · name the space · state the conclusion

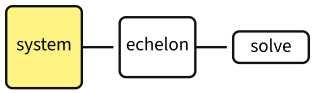
12 · Worked 1 · elimination

CH 1 · 4 STEPS

Solve x+2y−z=3; 2x+5y+z=9; −x−y+2z=0.

```
[1 2 -1 | 3]
[2 5  1 | 9]
[-1 -1 2 | 0]
```

1. $R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 + R_1$ give new rows (0,1,3|3) and (0,1,1|3).
2. $R_3 \leftarrow R_3 - R_2$ gives (0,0,−2|0), so z=0.
3. Back-substitute: y=3; then x=−3.
4. Answer (−3,3,0). Substitution into all three originals returns 3, 9 and 0.



Record reversible moves: the audit trail locates a sign error.

13 · Parametric answer

CH 2 · WHOLE FAMILY

For x+2y−z=4 and 2x+4y−2z=8, the second row is twice the first. No contradiction; y,z are free.

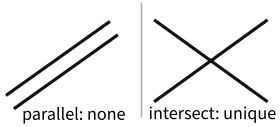
```
y=s, z=t ⇒ x=4-2s+t
(x,y,z)=(4,0,0)+s(-2,1,0)+t(1,0,1),
s,t∈R.
```

Check: each direction solves the homogeneous equation; the particular vector supplies 4. Never report only one convenient solution.

QUICK FAULT	WHY WRONG
one parameter for two free variables	misses independent directions
omits s,t∈R	parameter domain unstated
checks reduced row only	may miss transcription error

13b · Classify at a glance

CH 2 · GEOMETRY



Coincident lines give infinitely many solutions; distinct parallel lines give a contradiction.

LAST MATRIX SAYS	WRITE
contradiction	"inconsistent; solution set ∅"

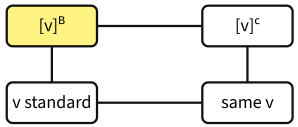
STOP RULE → *Echelon form is already enough for pivots, zero or contradiction rows and back-substitution. Reduce to RREF only if easier to read; extra operations add arithmetic-error risk.*

14 · Worked 2 · change basis

CH 9 · 4 STEPS

Given B={(1,0),(1,1)}, C={(1,1),(0,1)}, $[v]^B$=(2,3). Find $[v]^c$.

1. Reconstruct: v=2(1,0)+3(1,1)=(5,3).
2. Write (5,3)=a(1,1)+b(0,1)=(a,a+b).
3. a=5; a+b=3 ⇒ b=−2.
4. $[v]^c$=(5,−2). Check: 5(1,1)−2(0,1)=(5,3).



Both coordinate routes must reconstruct the same vector.

15 · Basis/space traps

CH 6–9 · RECALL

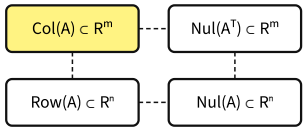
IF ASKED…	DO THIS
b in span?	solve Ac=b; reconstruct b
columns independent?	solve Ac=0; only trivial?
basis for Col(A)	original pivot columns
basis for Nul(A)	parameter direction vectors
coordinates	solve coefficients in declared order
one-to-one / onto	pivot every column / every row

Mini-check: B={(1,1),(1,−1)}, x=(5,1). Solve a+b=5, a−b=1 ⇒ a=3,b=2, so $[x]^B$=(3,2); reconstruct 3(1,1)+2(1,−1)=(5,1).

LANGUAGE → *Kernel vectors are inputs; image vectors are outputs. Codomain is declared; image is what is actually reached.*

15b · Four spaces snapshot

CH 7 · DIMENSIONS



Row/column spaces have dimension r; null spaces have dimensions n−r and m−r.

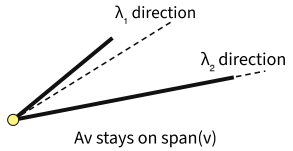
DIRECTION AUDIT → *Label each coordinate vector with its ordered basis. A basis-column matrix maps basis coordinates to standard; its inverse returns standard to basis. Reconstruct from both coefficient lists: the same standard vector must result. This catches arithmetic and a reversed matrix.*

16 · Worked 3 · diagonalise

CH 10 · 4 STEPS

A=[[4,1],[0,2]]. Diagonalise and state A^k.

1. Triangular entries give λ_1=4, λ_2=2.
2. λ=4: v_1=(1,0). λ=2: 2x+y=0, choose v_2=(1,−2).
3. P=[[1,1],[0,−2]], D=diag(4,2). Independent columns license A=PDP⁻¹.
4. A^k=PD^kP⁻¹, D^k=diag(4^k,2^k). Verify AP=PD.



Eigenvectors may stretch, shrink or reverse, but remain on their own spans.

17 · Markov drill

CH 11 · FIXED POINT

P=[[0.8,0.3],[0.2,0.7]] acts on column states. Columns sum to one. Solve Px=x with x_1+x_2=1:

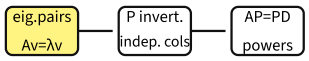
```
0.8x₁+0.3x₂=x₁ ⇒ 0.2x₁=0.3x₂ ⇒
x₁:x₂=3:2 ⇒ x=(0.6,0.4).
```

Check P(0.6,0.4)=(0.60,0.40), entries nonnegative, total 1. This proves stationarity only; do not overclaim convergence from every initial state.

EIGENVALUE |λ|	MODE IN A^k
>1	grows
<1	decays
=1	persists or oscillates

17b · Diagonalisation audit

CH 10–11



AP=PD checks every eigenpair and its alignment with D in one multiplication.

MULTIPLICITY	MEANING
algebraic	root count in det(A−λI)
geometric	dim ker(A−λI)
diagonalizable	sum of eigenspace dimensions = n

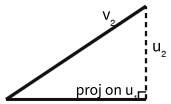
FAILURE CHECK → *A repeated root gives algebraic multiplicity, not the number of independent eigenvectors. Solve (A−λI)v=0 and count the eigenspace dimension before claiming diagonalisation. P needs n independent columns, each aligned with its λ in D. Test AP=PD: it checks all eigenpairs at once before powers are computed.*

18 · Worked 4 · Gram–Schmidt

CH 12 · 4 STEPS

v_1=(1,1,0), v_2=(1,0,1).

1. u_1=v_1=(1,1,0); $u_1 \cdot u_1$=2.
2. $\text{proj}_{u_1}(v_2)$=((\$v_2 \cdot u_1$)/($u_1 \cdot u_1$))$u_1$=(1/2)(1,1,0).
3. u_2=v_2−proj=(1/2,−1/2,1); $u_1 \cdot u_2$=0.
4. If orthonormal requested: e_1=$u_1/\sqrt{2}$; e_2=$u_2/\sqrt{(3/2)}$. Otherwise {u_1,u_2} is already an orthogonal basis for the same span.



remove projection → orthogonal residual

At each stage subtract projections onto every earlier u_i.

19 · Projection traps

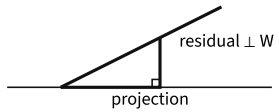
CH 4, 12 · DIAGNOSE

WRONG MOVE	CORRECTION
divide by ||u||	divide by u·u=||u||²
project onto u=0	undefined
orthogonal formula on non-orthogonal list	orthogonalise first or solve normal equations
subtract only latest projection	subtract every earlier projection
accept projection by arithmetic	check target membership + residual orthogonality

Cauchy–Schwarz: |⟨u,v⟩|≤||u||||v||; equality iff u,v dependent. An orthogonal list of nonzero vectors is automatically independent.

19b · Projection decision

CH 12 · BASIS TYPE



Closest-vector questions are projection questions: fitted point in W, residual in W⊥.

GIVEN TARGET BASIS	EFFICIENT MOVE
one nonzero u	single vector projection
orthogonal {u_i}	sum scaled projections
orthonormal {q_i}	sum ⟨v,q_i⟩q_i
general independent columns A	normal equations

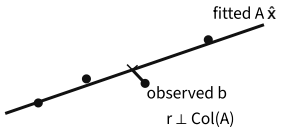
PROJECTION AUDIT → *Projection lies in the target subspace; its residual is orthogonal to every basis direction. In Gram–Schmidt test each new residual against every earlier u_i; normalise after checking the orthogonal list.*

20 · Worked 5 · least squares

CH 13 · 4 STEPS

Fit y≈a+bx to (0,1),(1,2),(2,2).

1. A has rows (1,0),(1,1),(1,2); b=(1,2,2). A^TA=[[3,3],[3,5]], A^Tb=(5,6).
2. Solve 3a+3b=5; 3a+5b=6. Subtract: 2b=1 ⇒ b=1/2.
3. 3a+3/2=5 ⇒ a=7/6. Fit: y=7/6+x/2.
4. Fitted values (7/6,5/3,13/6); residual b−A(a,b)=(−1/6,1/3,−1/6). Its dot products with (1,1,1) and (0,1,2) are both 0.



Normal equations encode the closest-point condition $A^T(b-Ax)$=0.

21 · Orthogonal · symmetric · SVD

CH 13 · COMPARE

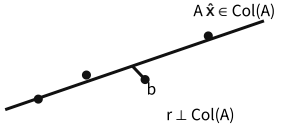
OBJECT	SIGNATURE	PAYOFF
orthogonal Q	Q^TQ=I	Q^{-1}=Q^T; preserves geometry
symmetric A	A^T=A	A=QDQ^T; real λ
SVD any real A	A=UΣV^T	rectangular/rank-deficient allowed

```
AᵀA vᵢ=σᵢ²vᵢ; σᵢ≥0. Right singular
vectors vᵢ are input directions;
left singular vectors uᵢ are output
directions.
```

CHECK → *Orthogonal diagonalisation is for real symmetric square matrices; SVD works for every real matrix.*

21b · Fit versus exact solve

CH 13 · RESIDUAL

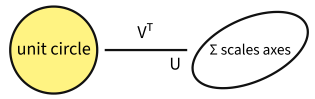


Least squares projects b onto Col(A); it does not make an inconsistent Ax=b exact.

DIAGNOSTIC	INTERPRETATION
r=0	b already in Col(A); exact solution
r≠0 but A^Tr=0	valid least-squares residual
A columns independent	least-squares coefficients unique
A rank deficient	fitted vector may be unique; coefficients need not be

22 · SVD picture · complex check

CH 13–14



SVD separates orthogonal input directions, scale, then orthogonal output directions.

COMPLEX ADJOINT

```
A*=Āᵀ · (AB)*=B*A*
Hermitian: H*=H. Unitary: U*U=I ⇒
U⁻¹=U*.
```

Source check: U=(1/$\sqrt{2}$)[[1,i],[i,1]]. Then U*=(1/$\sqrt{2}$)[[1,−i],[−i,1]]. In U*U, diagonal (1+(−i)i)/2=1 and off-diagonal (i−i)/2=0, hence U*U=I.

TRAP → *Ordinary transpose is the right adjoint only when entries are real. Conjugation makes ⟨v,v⟩ real and nonnegative.*

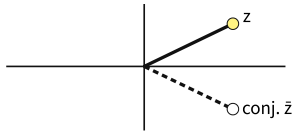
23 · Method chooser

TRIGGER → MOVE → PROOF

PROMPT TRIGGER	FIRST MOVE	FINAL CHECK
solve/classify Ax=b	row-reduce [A|b]	substitute; inspect contradiction/free vars
span / independence	Ac=b / Ac=0	reconstruct / trivial-only
basis / dimension	pivots + correct source vectors	span and independence; rank-nullity
linear map matrix	columns T(e_j)	direct substitution
change coordinates	reconstruct standard v	reconstruct destination

24 · Complex geometry

CH 14 · CONJUGATION



Conjugation reflects across the real axis and is built into the complex adjoint.

MATLAB AUDIT → *Use supported arithmetic or RREF to check a relation; then state the definition tested and conclusion justified. Output checks arithmetic, not proof; transpose replaces the adjoint only for real entries.*

Your exam allows one A4 sheet of notes, both sides. Check on the LMS whether printed notes are accepted; if they must be handwritten, copy it.

Forward this to your tutorial group

SHARED BY _____