MAST10006 Calculus 2 · Method Map

UNIVERSITY OF MELBOURNE · FIRST-YEAR UNDERGRADUATE · SEMESTER 2, 2026

Full guide + ask Sia about MAST10006

SIDE 1/2 EXAM MAP · LIMITS · SERIES · TAYLOR · HYPERBOLIC · INTEGRATION · ODEs · MULTIVARIABLE — ONE A4 NOTE SHEET — Mechanisms · decisions · formulas

0 · Exam Map

COURSE.JSON + BRIEF

Final written exam 60% | Test 20% | Tasks 20%

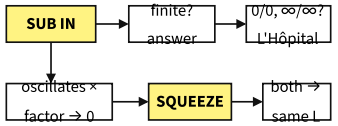
ASSESSMENT	WEIGHT	FORMAT
Final written exam	60%	3 h · in person · all content
Mid-semester test	20%	45 min
Assignments (best 6 of 7)	20%	one free miss

"Students are permitted 1 double-sided A4 page of notes. No other materials are permitted. Calculators are not permitted."

MAST10006 2026 SM1 Canvas subject summary + 2026 SM1 "Information about the exam" page – Semester 1 2026 offering; confirm this semester's rule on the LMS.

1 · Limits → Pick the Legal Move

LIMITS & CONTINUITY



Split only when each limit exists.

Limit triage: substitution decides whether a standard form, squeeze or L'Hôpital is legal.

EXISTENCE + STANDARDS

```
lim_{x→a} f exists ⇔ left limit = right limit.
lim_{x→0}(sin x)/x = 1 · lim(e^x-1)/x = 1 · lim(1-cos x)/x = 0.
```

RULE	CONDITIONS	EXAM CONCLUSION
Squeeze	g≤f≤h near a; g,h→same L	f→L
L'Hôpital	0/0 or ∞/∞; derivative quotient limit exists	lim f/g = lim f′/g′
Continuity	f(a) exists; limit exists; equal	continuous at a

FAST PATTERNS

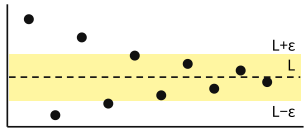
Manufacture a standard limit: for sin(kx)/x, insert kx: [sin(kx)/(kx)]·k. For $(e^{kx}-1)/x$, use $[e^{kx}-1]/(kx)\cdot k$. Rewrite tan as sin/cos before pairing factors.

Squeeze prototype: $-1\le\sin(1/x)\le 1$ gives $-x^2\le x^2\sin(1/x)\le x^2$, hence the middle tends to 0. If multiplying an inequality by a negative quantity, reverse its direction.

Piecewise joins: inspect boundaries only. Match left limit = right limit = assigned value. For differentiability, also match the one-sided derivatives. A removable break has equal one-sided limits but a missing/wrong value; a jump has unequal one-sided limits; an infinite break has a blow-up.

2 · Sequences & Series

SEQUENCES & SERIES



eventually every a_n lies inside L±ε

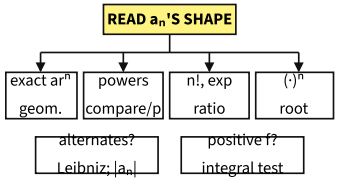
Convergence means the terms eventually remain in every band about one finite L.

SEQUENCE PROOF PATTERN

```
monotone + bounded ⇒ convergent. For a recurrence: prove direction; prove a bound; set L = lim a_n = lim a_{n+1}; solve; discard roots outside the bound.
```

SERIES: RUN THE FILTER FIRST

```
Σa_n converges ⇔ partial sums S_N = Σ_{1..N} a_n approach a finite S.
If lim a_n ≠ 0 or does not exist ⇒ diverges. If lim a_n=0: inconclusive.
```



Test selector. Ratio/root with L<1 converges, L>1 diverges, L=1 says nothing.

BENCHMARK	CONVERGES EXACTLY WHEN
Σar^n	$\lvert r\rvert<1$; sum = first term/(1−r)
$\Sigma 1/n^p$	p>1
$\Sigma(-1)^n b_n$	b_n decreases and $b_n\to 0$

TEST	CONDITIONS / DECISION	TYPICAL TRIGGER
Direct comparison	$0\le a_n\le b_n$ and Σb_n converges ⇒ Σa_n converges; $0\le b_n\le a_n$ and Σb_n diverges ⇒ Σa_n diverges	visible inequality
Limit comparison	$a_n,b_n>0$ and $a_n/b_n\to c$ with $0<c<\infty$ ⇒ same behaviour	rational powers; choose dominant p-series
Integral	f positive, continuous, decreasing; $\Sigma f(n)$ and $\int f$ have same behaviour	log/power expression
Ratio/root	L<1 converge absolutely; L>1 diverge; L=1 inconclusive	factorial/exponential or nth power

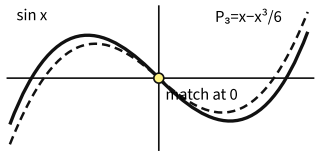
Conditional vs absolute: first test $\Sigma|a_n|$. If it converges, so does Σa_n absolutely. If $\Sigma|a_n|$ diverges but an alternating test makes Σa_n converge, convergence is conditional.

3 · Taylor / Maclaurin

TAYLOR SERIES

DEGREE N ABOUT A

```
P_n(x)=Σ_{k=0..n} f^(k)(a)(x-a)^k/k!
f(x)=P_n(x)+R_n(x).
```



P_3 matches sin x and its first three derivatives at the centre; the gap is R_3.

CATALOGUE	OPENING TERMS
e^x	$1+x+x^2/2!+x^3/3!+\cdots$
sin x	$x-x^3/3!+x^5/5!-\cdots$
cos x	$1-x^2/2!+x^4/4!-\cdots$
1/(1−x)	$1+x+x^2+x^3+\cdots$
log(1+x)	$x-x^2/2+x^3/3-\cdots$
$(1+x)^\alpha$	$1+\alpha x+\alpha(\alpha-1)x^2/2!+\cdots$

Build, do not re-derive: substitute → multiply → differentiate → integrate, within the radius of convergence.

LAGRANGE ERROR

```
R_n(x)=f^(n+1)(c)(x-a)^(n+1)/(n+1)!
|R_n|≤M|x-a|^(n+1)/(n+1)!, M bounds |f^(n+1)| on the whole interval.
```

Bounding recipe: identify degree n and centre a → differentiate once beyond the degree → bound that derivative on the entire interval between a and x → substitute M, |x−a| and (n+1)! → state the numerical guarantee, not merely the approximation.

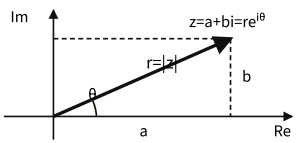
BUILD ARCTAN WITHOUT DERIVATIVE TABLES

```
1/(1-u)=Σu^n; put u=-x^2:
1/(1+x^2)=1-x^2+x^4-⋯.
Integrate; arctan 0=0 ⇒ arctan x=x-x^3/3+x^5/5-⋯.
```

OFF-BY-ONE → *Degree n means the next derivative, next power and (n+1)!.*

4 · Hyperbolic + Complex

HYPERBOLIC FUNCTIONS



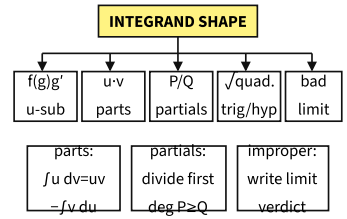
Euler: $e^{i\theta}=\cos\theta+i\sin\theta$; principal argument lies in $(-\pi,\pi]$.

```
sinh x=(e^x-e^-x)/2 · cosh x=(e^x+e^-x)/2
cosh^2x-sinh^2x=1 · (cosh x)′=+sinh x.
```

INVERSE	LOG FORM / DOMAIN
arcsinh x	$\log(x+\sqrt{(x^2+1)})$; all x
arccosh x	$\log(x+\sqrt{(x^2-1)})$; $x\ge 1$

5 · Integration: Shape → Tool

INTEGRATION TECHNIQUES



The exam skill is naming the method before doing algebra.

ROOT	SUBSTITUTION	IDENTITY
$\sqrt{(a^2-x^2)}$	x=a sin θ	$1-\sin^2=\cos^2$
$\sqrt{(a^2+x^2)}$	x=a tan θ or a sinh u	$1+\tan^2=\sec^2$; $1+\sinh^2=\cosh^2$
$\sqrt{(x^2-a^2)}$	x=a sec θ or a cosh u	$\sec^2-1=\tan^2$; $\cosh^2-1=\sinh^2$

Partial fractions. Repeated $(x-a)^2$ needs $A/(x-a)+B/(x-a)^2$. Irreducible quadratic needs full numerator Bx+C.

Powers. For sin/cos or sinh/cosh: peel one odd factor and use the identity; if both powers are even, use a double-angle identity.

PARTS + PARTIAL-FRACTION SAFEGUARDS

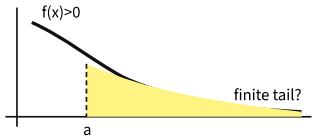
LIATE chooses u: Logarithm → Inverse-trig → Algebraic → Trig → Exponential. A lone log or arctan hides dv=dx. Parts is useful only if ∫v du is simpler; a single composite f(g)g′ is ordinary substitution.

Return-of-integral: $e^x\sin x$ or $e^x\cos x$ cycles. Use the same u-type twice; when I reappears, solve the equation for I and add +C once, at the end.

PARTIAL-FRACTION SETUP

```
deg P≥deg Q ⇒ long-divide first.
(x-a)(x-b): A/(x-a)+B/(x-b).
(x-a)^2: A/(x-a)+B/(x-a)^2.
x^2+px+q irreducible: (Bx+C)/(x^2+px+q).
```

Back-substitution audit: state the inverse-function range, recover the correct sign of every square root, include dx, and return the final indefinite answer to x.



Improper means a limit: $\int_a^\infty f\,dx=\lim_{t\to\infty}\int_a^t f\,dx$. Finite limit ⇔ convergent.

INTERIOR BLOW-UP → *Split at the bad point. The whole integral converges only if both one-sided integrals converge.*

6 · First-Order ODEs

FIRST-ORDER DIFFERENTIAL EQUATIONS

RECOGNISE	METHOD / TRANSFORMATION
y′=f(x)g(y)	separate: $\int dy/g(y)=\int f(x)dx+C$
y′+P(x)y=Q(x)	$\mu=e^{\int P dx}$; $(\mu y)'=\mu Q$
y′=F(y/x)	v=y/x; y=vx; y′=v+xv′
y′=F(ax+by+c)	u=ax+by+c; transform u′
$y'+Py=Qy^n$	Bernoulli: $u=y^{1-n}$ → linear

INTEGRATING-FACTOR RITUAL

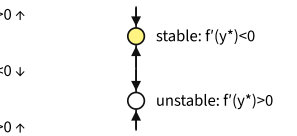
```
1 standardise coefficient of y′ to 1 → 2 read P,Q → 3 μ=e^∫Pdx → 4 (μy)′=μQ → 5 integrate → 6 divide by μ.
```

Separable warning: before dividing by g(y), record g(y)=0; test these constant equilibria in the original ODE.



solution curves stay tangent to y′=f(x,y)

A direction field gives behaviour without integration; isoclines join equal slopes.



Phase line: solve f(y)=0, mark signs between equilibria, then classify attraction or repulsion.

APPLICATIONS

```
logistic: P′=rP(1-P/K): equilibria 0 and K.
mixing: amount rate = rate in - rate out.
```

QUALITATIVE MARKS WITHOUT SOLVING

For autonomous y′=f(y), zeros of f are constant solutions. On each interval, f>0 sends y upward and f<0 downward. Arrows toward y* mean stable; away mean unstable. The derivative shortcut gives f′(y*)<0 stable and f′(y*)>0 unstable; f′(y*)=0 requires the sign diagram.

A direction field assigns slope f(x,y) at every point; draw short ticks on isoclines f=c, then thread a solution curve tangent to them. It gives trend and asymptotic behaviour, not exact times or values.

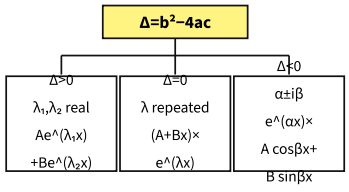
Application setup: in a mixing tank, "out" uses current concentration = amount/volume. In logistic growth, K is the carrying-capacity equilibrium and 0 is extinction; classify each from the phase line.

7 · Second-Order ODEs

SECOND-ORDER DIFFERENTIAL EQUATIONS

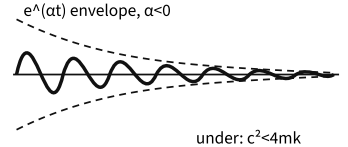
SPLIT THE PROBLEM

```
ay″+by′+cy=g(x) ⇒ y=y_h+y_p.
Homogeneous: aλ^2+bλ+c=0; Δ=b^2-4ac.
```



The discriminant is the only homogeneous-case decision; two conditions fix A and B.

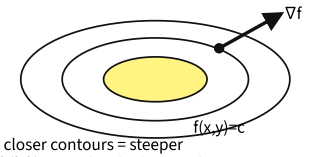
FORCING G(X)	TRIAL y_p
polynomial degree n	general polynomial degree n
e^{kx}	Ce^{kx}
cos ωx or sin ωx	C cos ωx+D sin ωx — both
trial overlaps y_h	multiply by x; by x^2 for repeated overlap



Free mass–spring: over-damped Δ>0; critical Δ=0; under-damped Δ<0.

8 · Multivariable Core

MULTIVARIABLE CALCULUS



$\nabla f=(f_x,f_y)$ is normal to a level curve and points to steepest ascent.

TANGENT PLANE + DIRECTION

```
z≈f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b).
D_û f=∇f·û; normalise û first.
```

2-D limits: two paths with different values prove non-existence. Agreement on every straight line does not prove existence; polar coordinates can establish a limit when the r→0 expression becomes independent of angle.

$D=F_{xx}F_{yy}-F_{xy}^2$	CLASSIFICATION
D>0, $f_{xx}>0$	local minimum
D>0, $f_{xx}<0$	local maximum

MAST10006 Calculus 2 · Worked Drills & Traps

UNIVERSITY OF MELBOURNE · CARRY THE METHOD TO THE VERDICT

Full guide + ask Sia about MAST10006

SIDE 2/2 · WHICH-IS-TRUE CONTRASTS · SHORT-ANSWER FRAMES · 8 SOURCE-WORKED APPLICATIONS · DRILL BY HAND · NO CALCULATOR · justify conditions · finish with a verdict

9 · Limits: Two Complete Answers

WORKED EXAMPLES

A · MANUFACTURE THE STANDARD FORM

Evaluate $\lim_{x\to 0}\sin(3x)/\tan(2x)$.

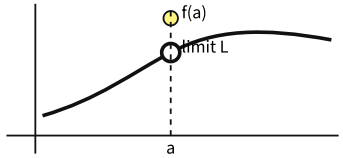
```
tan(2x)=sin(2x)/cos(2x)
sin(3x)/tan(2x)
=
[sin(3x)/(3x)]·[(2x)/sin(2x)]·(3/2)·cos(2x)
→1·1·(3/2)·1= 3/2 .
```

Frame: "Each factor has a limit, so the product law applies." Match sin(3x) with 3x, not x.

B · L'HÔPITAL, MARK BY MARK

Evaluate $\lim_{x\to 0}(e^x-1-x)/x^2$.

1. Substitution gives 0/0: rule is legal.
2. Differentiate separately: lim(e^x−1)/(2x).
3. Still 0/0: differentiate again → lim e^x/2.
4. At 0: 1/2 .



Removable discontinuity: the two-sided limit exists, but limit ≠ f(a).

WHICH IS TRUE?	VERDICT
A limit requires f(a) to exist.	**False.** It concerns nearby values.
Differentiable ⇒ continuous.	**True.** Converse fails: |x| at 0.
Outer squeeze bounds may differ.	**False.** They must approach the same L.

FOUR WAYS TO EARN THE JUSTIFICATION MARK

Limit law: name the law only after confirming the component limits exist; for a quotient the limiting denominator must be non-zero. **Standard form:** display the factor tending to 1. **Squeeze:** show both inequalities and the common outer limit. **L'Hôpital:** display the substituted 0/0 or ∞/∞ form before each pass.

CONTINUITY AT A JOIN X=A

```
1 f(a) exists.
2 lim_{x→a⁻} f = lim_{x→a⁺} f, so the two-sided
limit exists.
3 that common limit equals f(a).
For differentiability: also match the two
one-sided derivatives.
```

Squeeze mini-check: the wild factor may not have a limit. That is exactly why bounding works. What must have limits are the two outer functions, and those limits must be the same.

COMMON FALSE MOVES

10 · Series + Taylor Applications

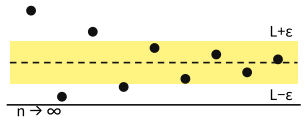
WORKED EXAMPLES

A · RATIO TEST: N!/N^N

Test $\Sigma_{n=1}^{\infty} n!/n^n$.

```
a_{n+1}/a_n=[(n+1)!/(n+1)^{n+1}]·[n^n/n!]
=n^n/(n+1)^n=[n/(n+1)]^n
=(1+1/n)^{-n}→e^{-1}<1.
Therefore the series converges
absolutely .
```

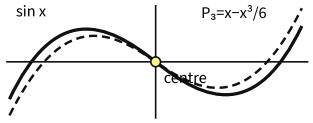
RATIO/ROOT LIMIT L	DECISION
L<1	converges absolutely
L>1	diverges
L=1	inconclusive: switch tests



A sequence converges only when its tail stays arbitrarily close to one finite L.

B · TAYLOR ERROR FOR E^0.2

```
P₂(x)=1+x+x²/2 ⇒
P₂(0.2)=1+0.2+0.02= 1.22 .
R₂=e^c x³/3!, 0<c<0.2. Since e^{0.2}<2, take
M=2.
|R₂(0.2)|≤(2/6)(0.008)≈2.7×10^{-3}.
Thus e^{0.2}=1.22±0.0027.
```



The polynomial matches locally; a Lagrange bound controls the gap away from the centre.

WHICH IS TRUE?	VERDICT
$a_n\to 0$ proves Σa_n converges.	**False:** harmonic counterexample.
Ratio L=1 proves divergence.	**False:** switch tests.
Absolute convergence implies convergence.	**True.**
R_2 uses f‴.	**False:** it uses f⁗ and x³.

TEST CONDITIONS: SAY THEM, THEN CONCLUDE

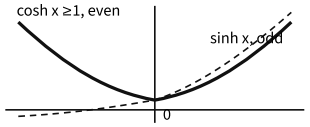
Alternating: write $a_n=(-1)^n b_n$, verify b_n decreases eventually and $b_n\to 0$. Then separately test $\Sigma|a_n|$ to decide absolute versus conditional. **Integral:** nominate f with f(n)=a_n and state positive, continuous, decreasing. **Comparison:** use non-negative terms and point the inequality toward a known convergent upper bound or divergent lower bound.

11 · Hyperbolic + Integration Drill

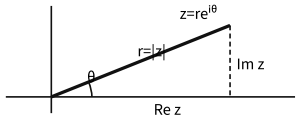
WORKED EXAMPLES

A · DERIVE ARCSINH X

1. Set y=arcsinh x, so x=sinh y=(e^y−e^{−y})/2.
2. Multiply by 2e^y: (e^y)²−2x(e^y)−1=0.
3. Quadratic formula: $e^y=x\pm\sqrt{(x^2+1)}$. Since $e^y>0$ and $\sqrt{(x^2+1)}>|x|$, choose +.
4. y=log(x+√(x²+1)) , all real x.



Both are exponential combinations; the master identity has a minus sign.



Polar form packages modulus and argument; Euler supplies the real and imaginary parts.

B · RETURN-OF-INTEGRAL

Evaluate I=∫e^x cos x dx by parts.

```
I=e^x sin x−∫e^x sin x dx.
∫e^x sin x dx=−e^x cos x+I.
I=e^x(sin x+cos x)−I ⇒ 2I=e^x(sin x+cos
x).
I=½e^x(sin x+cos x)+C.
```

RETURN LOOP	ACTION
I reappears after the second parts pass	substitute back, collect I terms, solve algebraically
Constant	add +C once, after solving for I

WHICH IS TRUE?	VERDICT
(cosh x)′=−sinh x.	**False:** +sinh x.
Take Re/Im before multiplying out.	**False:** extract at the end.
+C belongs after solving for I.	**True.**
An inverse-hyperbolic answer always includes its domain.	**True.**

SUBSTITUTION RANGE + BACK-SUBSTITUTION

For $\sqrt{(a^2-x^2)}$, x=a sin θ makes the root a cos θ only after choosing a range where cos θ≥0. For $\sqrt{(x^2+a^2)}$, x=a sinh u makes the root a cosh u. For $\sqrt{(x^2-a^2)}$, x=a cosh u makes it a sinh u. Transform dx, integrate, and return an indefinite integral to x.

COMPLEX-EXPONENTIAL RESULT TEMPLATE

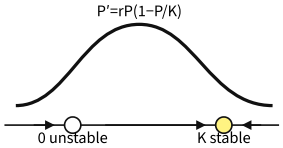
```
∫e^{ax}cos bx dx = Re[e^{(a+ib)x}/(a+ib)]+C.
∫e^{ax}sin bx dx = Im[e^{(a+ib)x}/(a+ib)]+C.
Use 1/(a+ib)=(a-ib)/(a²+b²), multiply
out, then select the part.
```

12 · First-Order ODE Drill

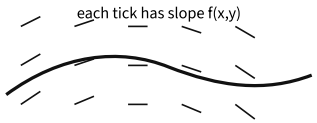
WORKED EXAMPLE

INTEGRATING FACTOR: XY′+Y=X²

1. **Standardise:** y′+y/x=x, so P=1/x, Q=x.
2. **Factor:** $\mu=e^{\int dx/x}=e^{\log x}=x$. No +C in the exponent.
3. **Fold:** multiply by x: (xy)′=x².
4. **Integrate/divide:** xy=x³/3+C, hence y=x²/3+C/x .



Logistic flow: 0 repels and K attracts when r and K are positive.



A solution curve must remain tangent to the local slope ticks.

SHORT-ANSWER FRAMES

PROMPT	WRITE THIS STRUCTURE
Separable IVP	record g(y)=0 → separate → integrate +C → use initial value → include valid equilibria
Linear ODE	standardise → state P,Q → μ → exact derivative → integrate → divide
Phase line	solve f(y)=0 → sign intervals → arrows → stable/unstable → long-run fate
Mixing model	amount rate = rate in − rate out; state concentration × flow rate

HOMOGENEOUS SUBSTITUTION

```
y′=F(y/x) → v=y/x, y=vx, y′=v+xv′ →
separate v and x → integrate → back-
substitute.
```

WHICH IS TRUE?	VERDICT
Read P before dividing by y″'s coefficient.	**False.** Standardise first.
Dividing by g(y) may lose equilibria.	**True.**
f′(y*)=0 proves stability.	**False:** inconclusive; inspect signs.

SUBSTITUTION CHECKLIST

For y′=F(y/x), set v=y/x so y=vx and y′=v+xv′; substituting v for y without transforming the derivative is invalid. For y′=F(ax+by+c), set u=ax+by+c and compute u′=a+by′ before substitution. For Bernoulli $y'+Py=Qy^n$, $u=y^{1-n}$ creates a linear equation in u.

13 · Second-Order ODE Drill

WORKED EXAMPLE

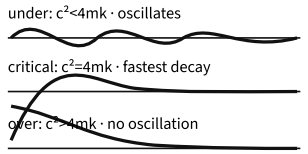
Solve y″+2y′+5y=0, y(0)=0, y′(0)=4.

```
λ²+2λ+5=0 ⇒ λ=(-2±√(4-20))/2= -1±2i .
y=e^{-x}(A cos2x+B sin2x).
y(0)=0 ⇒ A=0.
y′=Be^{-x}(2cos2x-sin2x).
y′(0)=4 ⇒ 2B=4 ⇒ B=2.
y=2e^{-x}sin2x.
```

Δ=b²−4ac

Δ>0	Δ=0	Δ<0
λ₁, λ₂	double λ	α±iβ
Ae^(λ₁x) +Be^(λ₂x)	(A+Bx)× e^(λx)	e^(αx)× cos/sin

Compute Δ first; it selects the only valid homogeneous form.



Damping is the same discriminant classification as the auxiliary quadratic.

PARTICULAR-SOLUTION FRAME

1. Solve the associated homogeneous equation for y_h.
2. Match a general trial to g(x): include every lower polynomial term and **both** sine/cosine terms.
3. If the trial is already in y_h, multiply by x; use x^2 for repeated overlap.
4. Substitute into the **original inhomogeneous** equation; solve coefficients.
5. Set $y=y_h+y_p$; apply two independent conditions last.

OVERLAP WITH y_H	MODIFIED TRIAL
no matching root	use the ordinary matched-shape trial
simple matching root	multiply the entire trial by x
repeated matching root	multiply the entire trial by x^2

WHICH IS TRUE?	VERDICT
Repeated root: (A+B)e^{λx}.	**False:** (A+Bx)e^{λx}.
Cos forcing needs only C cos.	**False:** add D sin.
Complex roots stay as complex exponentials.	**False:** convert to real cos/sin form.

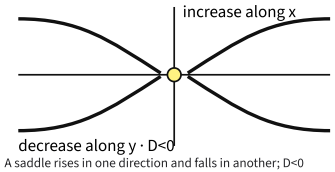
ROOT-CASE SELF-CHECK

14 · Multivariable Drill

WORKED EXAMPLE

Classify stationary points of f(x,y)=x³−3x+y².

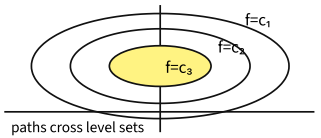
1. $f_x=3x^2-3=0 \Rightarrow x=\pm1$; $f_y=2y=0 \Rightarrow y=0$. Points: (1,0), (−1,0).
2. $f_{xx}=6x$, $f_{yy}=2$, $f_{xy}=0 \Rightarrow D=12x$.
3. At (1,0): D=12>0 and $f_{xx}=6>0 \Rightarrow$ local minimum .
4. At (−1,0): D=−12<0 ⇒ saddle .



A saddle rises in one direction and falls in another; D<0 regardless of f_{xx}.

PATH TEST: DISPROVE A 2-D LIMIT

For f(x,y)=xy/(x²+y²) at (0,0): path y=0 gives 0; path y=x gives x²/(2x²)=1/2. Different path limits ⇒ limit does not exist . Agreement on every line y=mx is not proof of existence; for a positive result, polar coordinates can show independence of θ as r→0.



Level curves are fixed-height sets in the domain; closer curves mean a steeper surface.

CHAIN RULE / DOUBLE INTEGRAL

```
If z=f(x(t),y(t)): dz/dt=f_x x′+f_y y′.
Over R=[a,b]×[c,d]: ∬_R f dA=∫_a^b ∫_c^d f(x,y)dy
dx. Inner first; hold the outer variable
constant.
```

WHICH IS TRUE?	VERDICT
∇f is tangent to a level curve.	**False:** perpendicular.
D=0 proves a saddle.	**False:** inconclusive.
Direction (3,4) may be used directly.	**False:** use û=(3/5,4/5).
Continuous mixed partials agree.	**True:** $f_{xy}=f_{yx}$.

TANGENT-PLANE APPLICATION

For f(x,y)=x²+2y² at (1,1): f(1,1)=3, $f_x=2x$ gives 2, $f_y=4y$ gives 4. Hence the plane is z=3+2(x−1)+4(y−1) . This is the first-order approximation near (1,1), not the exact surface.

Gradient/direction: normalise v to û=v/|v|, then $D_{\hat{u}}f=\nabla f\cdot\hat{u}$. Along a level curve the directional derivative is 0.

Semester 2 examination period, 2–20 November 2026.

Your exam allows one A4 sheet of notes, both sides. Check on the LMS whether printed notes are accepted; if they must be handwritten, copy it.