MAST10005 Calculus 1

THE UNIVERSITY OF MELBOURNE · SEMESTER 2, 2026

Full guide + ask Sia about MAST10005

SIDE 1 · WHAT THE FINAL REWARDS

EXAM REVISION SHEET · definitions → method → check

PRIORITY ORDER

1 · Exam map

COURSE FACTS

15 | 5 | 20 | 60%

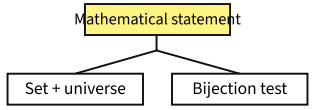
ASSESSMENT	WT
Six evenly weighted assignments	15%
Online 30-minute quiz	5%
In-person mid-semester test	20%
Three-hour written examination Held during the examination period	**60%**

Permitted materials: not confirmed for this offering in the course materials we hold — check the exam instructions on the LMS.

PRIORITY *The 60% final rewards controlled definitions, valid symbolic moves, preserved cases and verification in the original statement.*

2 · Logic, sets, functions

C1 · TOPIC MAP



Separate truth value, membership/universe and one-to-one/onto claims.

QUANTIFIER CONTROL

```
¬(∀x P(x)) ⇔ ∃x ¬P(x)
¬(∃x P(x)) ⇔ ∀x ¬P(x)
P ⇒ Q is not Q ⇒ P; contrapositive: ¬Q ⇒ ¬P.
```

IF THE CLAIM IS...	PROVE/CHECK
Universal	Arbitrary element; definition/theorem. One example never proves it.
Existential	Produce a valid witness in the declared domain.
False universal	One counterexample suffices.
Inverse exists	Injective *and* surjective; domain/codomain explicit.

First write the universe, domain and quantifiers. Then label each move as equivalence, one-way implication or theorem application.

SET + FUNCTION CONTROL

OBJECT	RECITE	CHECK IN WORKING
Set	Distinct objects treated as members of one object.	Membership and declared universe survive complement, union and intersection.
Injection	Equal outputs force equal inputs.	Start with $f(x_1)=f(x_2)$; prove $x_1=x_2$.
Surjection	Every codomain element is hit.	Take arbitrary y in codomain; solve f(x)=y with x in domain.
Bijection	Injection + surjection.	Only then claim an inverse pairing every codomain element with exactly one domain element.

PROOF ROUTE SELECTOR

3 · Complex numbers

C2 · PP1–7



Choose the representation that makes the operation visible.

Complex number $z=a+bi$, $a,b\in\mathbb{R}$, $i^2=-1$. ***Modulus*** is non-negative distance from the origin; ***argument*** locates nonzero z modulo full rotations.

CARTESIAN ⇔ POLAR

```
z = a + bi = r(cos θ + i sin θ)
r = |z| = √(a²+b²),  θ = arg z (mod 2π)
z̄ = a - bi,  z z̄ = |z|².
```

MULTIPLY / DIVIDE / POWERS

```
r₁cis θ₁ · r₂cis θ₂ = r₁r₂ cis(θ₁+θ₂)
(r₁cis θ₁)/(r₂cis θ₂) =
(r₁/r₂)cis(θ₁-θ₂), r₂≠0
(r cis θ)ⁿ = rⁿ cis(nθ).
```

ALL NTH ROOTS

```
If zⁿ = R cis θ, then
zₖ = R^{1/n} cis((θ+2πk)/n), k=0,…,n-1.
```

SEE	USE	CHECK
Add/subtract	Cartesian	Real/imaginary parts
Products, powers, roots	Polar	Modulus + every branch
Polynomial roots	FTA + branch list	Substitute in original

TRAP *The principal argument is one representative, not the only admissible angle.* *List every branch.*

OPERATION LEDGER

MOVE	RADIAL EFFECT	ANGULAR EFFECT
Conjugate	Modulus unchanged	Reflect argument across real axis
Multiply	Multiply moduli	Add arguments
Divide	Divide moduli; divisor nonzero	Subtract arguments
Power n	Raise modulus to n	Multiply argument by n
nth roots	Take nth root	Divide Θ+2πk by n

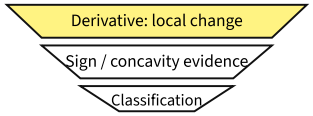
ROOT-EXTRACTION ROUTINE

1. Put the target in polar form and declare the argument family.
2. Take the required root of the modulus.
3. Distribute all admissible angles using k=0,…,n−1.
4. Remove no branch merely because it lies outside the principal range; rewrite an equivalent angle if useful.
5. Plot or compare spacing as a reasonableness check.
6. Substitute every candidate in the original polynomial.

POLYNOMIAL-ROOT CONTROL

4 · Derivatives & shape

C3 · PP1–5



Zero values create candidates; sign changes make classifications.

Derivative: the limit of an average rate of change, when it exists, describing local linear change.

DEFINITION + LOCAL LINEAR READING

```
f'(x) = lim_{h→0} [f(x+h)-f(x)]/h
f(x+h) ≈ f(x)+f'(x)h.
```

STRUCTURE	RULE
Sum / scalar	(af+bg)' = af'+bg'
Product	(fg)' = f'g+fg'
Composite	(f∘g)' = f'(g)g'
Implicit curve	Differentiate both sides in x; retain y' on dependent terms.

SKETCHING DECISION RULES

EVIDENCE	CONCLUDE
f' > 0 / f' < 0	Increasing / decreasing interval
f' changes +→− / −→+	Local maximum / minimum
f'' > 0 / f'' < 0	Concave up / down
Concavity changes at defined point	Inflection point

f'=0 is a candidate condition, not a classification.

Include endpoints and points where derivatives fail when the domain requires them.

CHECK *Predict sign and curve behaviour before algebra; test the classification against the sign chart after.*

CURVE-EVIDENCE WORKFLOW

1. Declare the domain; mark endpoints and places where the function or derivative is undefined.
2. Find candidate points from f'=0 and derivative failure.
3. Partition the domain; determine the sign of f' on each interval.
4. Classify only from the evidence: increasing/decreasing and any sign change.
5. For concavity, track the change of f' or the sign of f''; an inflection point must be defined and must change concavity.
6. Sketch only after intercept, domain and sign information agree.

IMPLICIT DIFFERENTIATION

Before	Identify which variable depends on x and which relation defines the curve.
During	Differentiate both sides with respect to x; dependent terms acquire y' through the chain rule.
After	Collect y' terms, solve only where the divisor is nonzero, and interpret the local slope on the original curve.

5 · Integrals: choose by structure

C3 · PP6–9

Definite integral: a limit of signed sums representing accumulated change over an interval. It combines signed contributions, not unsigned area by default.

ACCUMULATION / ENDPOINTS

```
If F'=f, then ∫ₐᵇ f(x)dx = F(b)-F(a).
Differentiate a proposed antiderivative
to verify it.
```

PATTERN IN INTEGRAND	CHOOSE	CONTROL
Composite with inner derivative	**Substitution**	Set u=g(x); rewrite dx and bounds, or return to x.
Product; one factor simplifies when differentiated	**Parts**	Keep the boundary term visible.
Rational function	**Partial fractions**	Factor denominator; include repeated factors; reconstruct first.
Known rate over interval	**Definite integral**	Interpret signed net change.

SUBSTITUTION

```
u=g(x), du=g'(x)dx
∫ f(g(x))g'(x)dx = ∫ f(u)du.
```

INTEGRATION BY PARTS

```
∫u dv = uv - ∫v du
∫ₐᵇ u dv = [uv]ₐᵇ - ∫ₐᵇ v du.
```

PARTIAL FRACTIONS SCAFFOLD

```
Factor denominator → write simple terms
→ solve coefficients → reconstruct →
integrate → differentiate check.
```

METHOD AUDIT

1. Name the structure, not the technique you happen to remember.
2. State the domain and any nonzero divisor.
3. Carry bounds/constants consistently.
4. Differentiate the result; for definite integrals also interpret sign.

TRAP *Do not use parts when substitution exposes the antiderivative directly.* *Structure licenses the method.*

DEFINITE-INTEGRAL READING

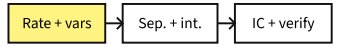
QUESTION	CONTROL
What is accumulated?	Name the instantaneous rate or signed contribution and its variable.
Which interval?	Carry lower and upper bounds through the method.
Net change or area?	A definite integral is signed accumulation; do not make every contribution positive by default.
Endpoint link?	Use an antiderivative and evaluate upper minus lower.
Does the answer fit?	Check sign, scale, domain and the derivative of the antiderivative.

PARTIAL-FRACTION CONTROL

1. Confirm the expression is rational and make it proper before decomposition.

6 · Differential equations

C4 · PP1–6



Declare → preserve exceptional cases → solve → select → substitute back.

Differential equation relates an unknown function to derivatives. ***Separable equation*** can place each variable and its differential on one side. An ***initial condition*** selects one solution from the family.

SEPARABLE FORM

```
dy/dx = g(x)h(y)
First record solutions with h(y)=0.
Where h(y)≠0: ∫dy/h(y) = ∫g(x)dx + C.
```

GROWTH / DECAY STRUCTURE

```
Rate proportional to quantity: dy/dx = ky.
Sign and units belong to the model;
preserve y=0 before division.
```

STEP	WHAT EARNS CONTROL
Translate	Name dependent/independent variables, units and rate relation.
Recognise	Can all y-terms and x-terms be isolated on valid domains?
Separate	Record equilibrium cases before dividing.
Integrate	Keep the constant and an implicit family if needed.
Select	Use the given initial value.
Verify	Substitute into both sides and check the condition.

Solving the equation does not validate the modelling assumptions.

TRAP *Dividing by the dependent variable can discard the zero solution; reporting a family without the initial condition leaves the task unfinished.*

EQUATION VERSUS MODEL

LAYER	QUESTION TO ANSWER
Translation	What quantity changes, with respect to what variable, and with what units/sign?
Equation type	Can the rate be written as a product of one-variable factors?
Algebra	Which zero factors or domains would a division remove?
Family	What constant remains after integrating?
Particular solution	Which initial value fixes that constant?
Verification	Does substitution satisfy the rate equation and initial value?
Interpretation	Do sign, units and stated assumptions support the modelled situation?

EXCEPTIONAL-CASE DISCIPLINE

```
Before dividing by h(y), solve h(y)=0.
Carry those equilibrium solutions
beside the separated family and test
the initial condition against both.
```

7 · Vectors & curves

C5 · PP1–7



Keep vector, scalar and curve-domain roles distinct.

Vector: magnitude and direction represented by ordered components. Equality is componentwise. ***Parametric curve***: coordinates are functions of one parameter.

LENGTH, ANGLE, ORTHOGONALITY

```
u·v = Σuᵢvᵢ = ‖u‖‖v‖cos θ
‖u‖ = √(u·u);  u ⊥ v ⇔ u·v=0.
Angle formula requires nonzero vectors.
```

PROJECTION STRUCTURE

```
proj_v u = (u·v / v·v)v,  v≠0.
Result is a vector parallel to v;
coefficient is scalar.
```

VECTOR-VALUED MOTION

```
r(t)=(x(t),y(t));  velocity v(t)=r'(t)
speed = ‖r'(t)‖;  stationary ⇔ r'(t)=0
vector.
```

QUESTION	DECISION
Same vector?	Compare every component in same coordinates.
Angle / perpendicular?	Scalar product; check nonzero vectors for angle.
Component along direction?	Projection; denominator v·v requires v≠0.
Tangent / motion?	Differentiate each coordinate; retain parameter interval.
Stationary?	Every component derivative zero at same t.

Speed is magnitude; velocity has direction. A parameterisation carries orientation and rate as well as geometric shape.

CHECK *State the coordinate system, preserve scalar/vector types, then interpret the geometry.*

GEOMETRY-TO-ALGEBRA ROUTINE

1. Write vectors in a common coordinate system and preserve component order.
2. For length, angle or orthogonality, compute the scalar product and state whether either vector may be zero.
3. For projection, name the target direction; the answer is parallel to it and the divisor requires a nonzero target vector.
4. For a plane curve, state r(t) and its parameter interval; different parameterisations can trace the same shape with different rate or orientation.
5. Differentiate componentwise. Interpret r'(t) as tangent/velocity, then take its magnitude only if speed is requested.

TYPE LEDGER

EXPRESSION	TYPE	MEANING/CHECK
u, v, r(t)	Vector	Components in chosen coordinates
u·v	Scalar	Angle/length/orthogonality relation

MAST10005 Calculus 1 · Self-test drills

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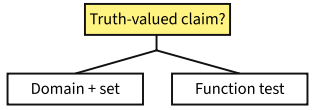
SIDE 2 · DRILLS & TRAPS

SELF-TEST · answer before reading the path

5 WORKED DRILLS

8 · Drill: proof repair

C1 · WORKED



The earliest unsupported move controls every dependent line.

PROMPT

A proposed proof switches a universal statement to a single example, reverses an implication and uses a function value outside its declared domain. Decide what can be concluded and identify the first condition that changes it.

ANSWER PATH	WRITE
1 · Declare	Domain, objects, truth-valued statement and quantifiers.
2 · Locate	Mark the first line where the premise no longer entails the next.
3 · Repair	Use the set/universe and label implication direction.
4 · Verify	Check any inverse claim is injective and surjective; test result in original statement.

VERDICT

A single example may disprove a universal statement but cannot establish one. P⇒Q does not license Q⇒P. An out-of-domain function value is inadmissible.

IF YOU SEE X → DO Y

"For every…"	Take an arbitrary admissible element.
"There exists…"	Give one valid witness.
Inverse claimed	Test injection and surjection separately.
Implication reversed	Use the contrapositive or prove the converse anew.

SELF-CHECK *Repair the first failure, not merely the final sentence.*

RECITE BEFORE CHECKING

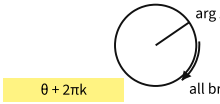
TERM	EXACT ROLE IN THE DRILL
Mathematical statement	A sentence or expression with an unambiguous truth value. First separate it from an open condition.
Set	A collection of distinct objects considered as members of one mathematical object. Keep the declared universe visible.
Bijection	A function both injective and surjective, pairing each codomain element with exactly one domain element.

FIVE-SENTENCE ANSWER SCAFFOLD

1. **Claim:** restate the proposition with quantifiers.
2. **Hypotheses:** give domain, codomain and membership.
3. **Route:** name direct, contrapositive or contradiction.
4. **Failure:** mark the first unsupported step.
5. **Repair:** replace it and recheck the original claim.

9 · Drill: missing complex roots

C2 · WORKED



Full rotations represent the same nonzero complex number.

PROMPT

A calculation treats the principal argument as the only possible angle and therefore omits valid roots of a complex polynomial. Decide what failed and repair the method.

ANSWER PATH	WRITE
1 · Represent	Choose Cartesian for addition or polar for products, powers and roots.
2 · Track	Keep modulus non-negative; write argument modulo a full turn.
3 · Distribute	Use (Θ+2πk)/n across every distinct k.
4 · Verify	Substitute each candidate root into the original polynomial.

VERDICT

The principal argument supplies a representative angle, not the full root set. Omitting k omits solutions.

BRANCH AUDIT

Correct count?	k=0,…,n−1 gives n distinct nth roots when the target is nonzero.
Correct radius?	Use the nth root of the target modulus.
Correct spacing?	Angles differ by 2π/n.
Correct claim?	Return every root to the original equation.

TRAP *A plausible diagram does not replace branch enumeration.*

RECITE BEFORE CALCULATING

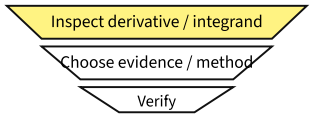
TERM	EXACT ROLE IN THE DRILL
Complex number	a + bi records real and imaginary coordinates with $i^2=-1$.
Modulus	Non-negative distance from the origin; controls radial size.
Argument	Angle of a nonzero complex number, defined up to full rotations.

REPRESENTATION TRANSFER TESTS

CHANGE	WHAT MUST CHANGE?
Addition replaces multiplication.	Return to Cartesian coordinates; angle addition is no longer the direct operation.
Target modulus changes.	Recompute the root radius but keep the full branch mechanism.
Target argument is rewritten by a full turn.	The root set is unchanged after the k-values are distributed correctly.
A proposed root fails substitution.	Repair the earliest modulus, angle or branch step; do not discard the check.

10 · Drill: calculus choices

C3 · WORKED



Classification and technique both require structural evidence.

PROMPT

A student differentiates correctly but labels every stationary point an extremum, then applies integration by parts where substitution exposes the antiderivative directly. Repair both conclusions.

ANSWER PATH	WRITE
1 · Stationary	f′=0 gives a candidate; make a sign chart or use valid concavity evidence.
2 · Classify	Require the relevant sign change; check domain and endpoints.
3 · Integrand	Look for f(g(x))g′(x) before choosing parts.
4 · Verify	Differentiate the antiderivative and compare with the original integrand.

VERDICT

Zero derivative alone does not prove maximum or minimum. A visible composite derivative licenses substitution; familiarity does not license parts.

JUDGEMENT CARDS

f′ changes +→−	Local maximum
f′ changes −→+	Local minimum
f″=0 only	Not enough: test concavity change.
Inner derivative present	Substitute; transform bounds or return to x.
Product simplifies by differentiating one factor	Use parts; keep boundary term.

TRAP *Candidate ≠ classification; pattern ≠ proof.*

TWO INDEPENDENT REPAIRS

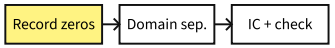
LINE UNDER REVIEW	WHY INSUFFICIENT	REQUIRED EVIDENCE
"f′(c)=0, so c is an extremum."	Only a stationary candidate.	Sign change, valid concavity evidence and domain/endpoints.
"The integrand is a product, so use parts."	Surface form does not select a method.	Inspect for an inner function and its derivative first.
"f″(c)=0, so c is an inflection."	Not a concavity change.	Function defined at c; concavity changes across it.

VERIFICATION LADDER

1. Compare each curve classification with its sign evidence and the original domain.
2. Differentiate every proposed antiderivative; for a definite integral also interpret upper minus lower as signed net change.

11 · Drill: lost ODE solution

C4 · WORKED



Every division creates a domain checkpoint.

PROMPT

A growth model divides by the dependent variable without checking the zero solution and reports a family of curves without using the initial condition. Find the first loss and finish the solution path.

ANSWER PATH	WRITE
1 · Declare	Variables, units, sign and rate relation.
2 · Preserve	Before division, solve the factor-equals-zero case.
3 · Separate	State the domain on which the division is valid; integrate both sides.
4 · Select	Apply the initial condition to fix the constant.
5 · Verify	Substitute into the differential equation and the condition.

VERDICT

Algebraic separation can discard exceptional solutions. A family is not the particular solution requested by an initial condition.

IF YOU SEE X → DO Y

dy/dx=g(x)h(y)	Test h(y)=0, then separate elsewhere.
"Rate proportional…"	Define the sign and units of the proportionality constant.
Initial value supplied	Use it before reporting the answer.
Proposed solution	Differentiate and substitute into both sides.

TRAP *A correct symbolic solution does not establish that the model fits the situation.*

RECITE THE THREE OBJECTS

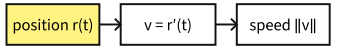
OBJECT	WHAT IT CONTROLS
Differential equation	Unknown function, independent variable and their rate relation.
Separable equation	Whether variables and differentials can be placed on opposite sides on a declared domain.
Initial condition	Which member of an integrated family is the particular solution.

TRANSFER: CHANGE ONE PREMISE

CHANGED CONDITION	FIRST LINE TO REVISIT
The initial value is an equilibrium.	The exceptional solution recorded before division may be the selected solution.
The separated divisor vanishes.	Restrict the separated derivation to a valid domain and keep the excluded case separate.
The proportionality sign changes.	Reinterpret growth versus decay and check the model's stated sign.

12 · Drill: vector-valued motion

C5 · WORKED



Vector differentiation first; scalar magnitude second.

PROMPT

A parametric curve has a valid geometric path, but the working confuses velocity with speed and treats a zero component derivative as a stationary point. Repair the interpretation.

ANSWER PATH	WRITE
1 · Declare	Coordinate system, parameter interval and vector-valued position.
2 · Differentiate	Compute every component of r′(t).
3 · Type	Velocity r′(t) is a vector; speed ‖r′(t)‖ is a non-negative scalar.
4 · Stationary	Require the entire velocity vector to be zero at the same parameter value.
5 · Interpret	Retain orientation, rate and geometry; verify on the declared interval.

VERDICT

One zero component means motion has no component in that coordinate direction at that instant; it does not make the whole curve stationary.

TYPE-AND-DOMAIN AUDIT

u·v	Scalar; supports angle, length and orthogonality.
$\text{proj}_v u$	Vector parallel to nonzero v.
r′(t)	Vector tangent/velocity; orientation retained.
‖r′(t)‖	Scalar speed; never negative.
Same geometric path	May still have different orientation or rate.

TRAP *Keep vector and scalar types visible on every line.*

RECITE THE DEFINITIONS

TERM	DEFINITION/USE
Vector	An object with magnitude and direction represented by ordered components in a chosen coordinate system.
Scalar product	The componentwise product sum connecting vectors to length, angle and orthogonality.
Parametric curve	A curve whose coordinates are functions of a common parameter.

TRANSFER: SAME PATH, CHANGED MOTION

OBSERVATION	SAFE CONCLUSION
Two parameterisations have the same image.	They share geometric shape only; compare intervals, orientation and rate separately.
x′(t)=0 but y′(t)≠0.	Velocity is not zero; the tangent direction has no x-component at that instant.

13 · Fast method chooser

ALL CHAPTERS

IF YOU SEE…	DO…	THEN CHECK…
Universal claim	Arbitrary admissible element	Every implication
Suspected false universal	Counterexample	Domain membership
Inverse function	Injection + surjection	Domain/codomain
Complex products/roots	Polar form	All argument branches
Stationary candidate	Sign/concavity evidence	Actual change
Composite integrand	Substitution	Bounds/variable
Simplifying product	Parts	Boundary term
Rational integrand	Factor + partial fractions	Reconstruction
Separable rate	Preserve equilibria; separate	Initial condition
Angle/projection	Scalar product	Nonzero divisor
Parametric motion	Differentiate components	Vector vs scalar

14 · Classic traps

FAQ + LIMITS

TEMPTATION	REPAIR
Example "proves" ∀	Use it only to disprove, or prove the arbitrary case.
Converse assumed	Keep implication direction explicit.
Principal arg only	Add 2πk before dividing by n.
f′=0 ⇒ extremum	Test sign change/classification evidence.
f″=0 ⇒ inflection	Require concavity change and defined point.
Technique by familiarity	Name integrand structure first.
Divide by y	Record y=0 or other equilibrium first.
Family reported	Apply the initial condition.
One component zero	Stationary needs the zero vector.
Speed = velocity	Speed is ‖velocity‖.

15 · Exam-morning plan

STUDY STRATEGY

1. **Retrieve:** say each definition and theorem condition without notes.
2. **Rebuild:** reproduce one method from its hypotheses, not from a memorised surface pattern.
3. **Change:** alter one premise and predict the first equality or implication that fails.
4. **Verify:** return every result to the original statement, equation, polynomial or curve.
5. **Prioritise:** revisit repeated errors and their exceptional cases.

CLOSED-NOTE RECALL CIRCUIT

WHEN STUCK