MTH2010 Multivariable Calculus

MONASH UNIVERSITY · LEVEL 2 · SEMESTER 2, 2026 · EXAM REVISION SHEET · SIDE 1 OF 2

Full guide + ask Sia about MTH2010

SIDE 1/2 DEFINITIONS → FORMULAS → DECISIONS → CHECKS **FINAL EXAMINATION 50%** v2 · 2026-10-25

1 · Exam Map

COURSE.JSON · ASSESSMENT

ACQ | SQ 10 | MST I 15 | MST II 15 | FINAL 50%

COMPONENT	WEIGHT	FORMAT/SOURCE FACT
Applied class quizzes	10%	11 × 1%; best 10
Seminar quizzes	10%	5 × 2%
Mid-semester I	15%	Week 4, in class
Mid-semester II	15%	Week 9, in class
Final examination	50%	Final assessment period

Permitted materials: not confirmed for this offering in the course materials we hold — check the exam instructions on the LMS. *Source: BRIEF.md; course.json.*

Two practice finals with solutions and a final exam formula sheet are posted in the Week 11 section on Moodle. Use them to rehearse **setup, theorem choice and checking**.

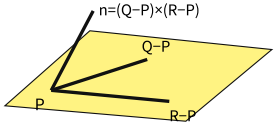
2 · Vectors, Lines, Planes

CH 1 · METHOD

CORE GEOMETRY

```
a·b = |a||b|cosθ; a⊥b ⇔ a·b=0
a×b ⊥ a,b; |a×b| = parallelogram area
Line: r=r₀+td   Plane: n·(r-r₀)=0
```

IF ASKED	DO
Plane through P,Q,R	u=Q−P, v=R−P, n=u×v; point-normal form; substitute all 3 points.
Line vs plane	Compute d·n. Non-zero ⇒ one intersection. Zero ⇒ parallel; substitute a line point to separate contained/disjoint.
Tangent to r(t)	Find t_0 first; line through $r(t_0)$ with direction $r'(t_0)$.



Plane from three points: two edge vectors determine the normal. Source figure: Ch 1.

GEOMETRY CHECKS

Angle: cosθ=(a·b)/(|a||b|); keep θ in [0,π]. **Volume:** |a·(b×c)|. A symmetric line form omits any coordinate whose direction component is zero; do not divide by zero.

For two intersecting curves, solve their parameters separately, confirm the same meeting point, then compute the angle from their evaluated tangent vectors. A regular parametrisation has r'(t)≠0.

3 · Limits & Continuity

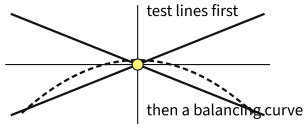
CH 2 · DECISIONS

GOAL	FAST ROUTE	WHAT EARNS THE CONCLUSION
Disprove limit	Axes → y=mx → degree-balancing curve	Two paths give different limits.
Prove limit	Limit laws; squeeze; polar	A bound valid for *all* nearby points.
Continuously extend	Find the limit first	Define f(a,b)=L only if L exists.

POLAR PROOF PATTERN

```
x=r cosθ, y=r sinθ. Show |f(r,θ)-L| ≤ g(r) for every θ, with g(r)→0. The bound must be uniform in θ.
```

Domain: logs need positive arguments; even roots need non-negative arguments; denominators must be non-zero. **Range/levels:** solve f=c and list ellipse/curve, point and empty cases separately.



Path test: one path can disprove; agreement on chosen paths never proves. Source figure: Ch 2.

4 · Partials, Plane, Linearisation

CH 3 · FORMULA

AT (A,B)

```
Tangent plane / linearisation:
L(x,y)=f(a,b)+fₓ(a,b)(x-a)+f_y(a,b)(y-b)
```

- Hold every other variable fixed for a partial. At a piecewise seam, write the **difference quotient** and check one-sided limits if needed.
- Continuous partials on a disc around the point ⇒ differentiable ⇒ continuous. Existing partials alone do *not* imply either.
- Mixed partials agree when continuous nearby (Clairaut). In the course notation, read differentiation order from the right.
- PDE verification: compute exactly the listed partials, substitute, simplify both sides.

CHECK *Substitute the base point into L; justify an estimate by differentiability, not by proximity alone.*

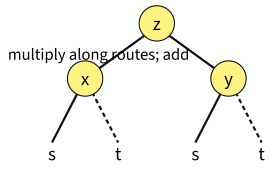
Definition audit: tangent-plane work needs f(a,b), both first partials and the displacements. For a partial at a seam use [f(a + h,b)−f(a,b)]/h or its y-analogue. If mixed partials are not known continuous nearby, do not swap their order merely because the notation looks symmetric.

5 · Chain Rule & Implicit

CH 4 · HIGH YIELD

TWO ROUTES TO S

```
∂z/∂s = fₓ(x,y)·∂x/∂s + f_y(x,y)·∂y/∂s
Outer partials at the inner point; inner partials at (s,t).
```



Dependency tree: every route to the chosen variable contributes one product. Source figure: Ch 4.

Implicit function checklist: point satisfies F=0; relevant partials continuous nearby; dependent-variable partial is non-zero.

IMPLICIT DERIVATIVES

```
F(x,y)=0: dy/dx = -Fₓ/F_y
F(x,y,z)=0: z_x = -Fₓ/F_z, z_y = -F_y/F_z
```

6 · Gradient & Direction

CH 4 · RECALL

```
∇f=(fₓ,f_y,f_z)   D_u f=∇f·u, |u⊨1
max rate=|∇f| along ∇f; steepest descent along -∇f
```

OBJECT	NORMAL / TANGENT RULE
Level surface F=c	∇F(p) is normal; plane ∇F(p)·(r−p)=0.
Intersection F=c, G=d	Tangent direction ∇F(p)×∇G(p).
Given direction v	Normalise first: u=v/|v|.

TRAP *A raw direction vector gives a scaled rate. Write the norm before the dot product.*

10 · Line Integrals

CH 9 · DISTINGUISH

INTEGRAL	FORMULA / ORIENTATION
Scalar	$\int_C f\,ds=\int_a^b f(r(t))\lvert r'(t)\rvert dt$; direction-independent.
Vector/work	$\int_C F\cdot dr=\int_a^b F(r(t))\cdot r'(t)dt$; reverse direction ⇒ reverse sign.

For corners, split into smooth pieces with separate parametrisations, intervals and arrows; add the pieces.

Field sketch check: predict the sign of F·r' before integrating. Tangential alignment gives positive work; opposition gives negative work; perpendicularity gives zero contribution.

7 · Optimisation

CH 5 · DECISION TABLE

Critical point: ∇f=0 or a partial fails to exist. Compute $A=f_{xx}$, $B=f_{xy}$, $C=f_{yy}$ and $D=AC-B^2$ *at each candidate*.

CONDITION	CLASSIFICATION
D>0, A>0	Local minimum
D>0, A<0	Local maximum
D<0	Saddle
D=0	Inconclusive: inspect directly



Contours distinguish minimum, maximum and saddle. Source figure: Ch 5.

ABSOLUTE EXTREMA ON A REGION

1. Confirm continuous on a **closed, bounded** set.
2. Find interior critical points.
3. Restrict to every boundary piece; find its critical points.
4. Add corners/endpoints; compare *all* values.

CONSTRAINED

```
∇f=λ∇g together with g=0. Two constraints ⇒ ∇f=λ∇g+μ∇h.
```

Lagrange candidates are not automatically absolute on an unbounded constraint: inspect behaviour at the ends. Follow the named method when a question requests multipliers.

11 · Surfaces & Flux

CH 11 · METHOD

```
r(u,v): dS=|r_u×r_v|du dv
Flux=∬F(r(u,v))·(r_u×r_v)du dv
```

Reverse the cross-product order to match upward/downward/outward/inward. For z=g(x,y): $dS=\sqrt{(1+g_x^2+g_y^2)}\,dA$ and the upward vector element is $(-g_x,-g_y,1)dA$.

QUANTITY	ORIENTATION?
Area / scalar surface integral	No; use magnitude
Flux	Yes; reversing normal negates

For a graph, verify the chosen vector element has the requested sign in its third component. For a closed surface, "outward" is the default in the course material. For a piecewise closed surface, give each piece its own outward normal.

CROSS-PRODUCT CHECK *Compute r_u, r_v and $r_u\times r_v$ slowly; verify it is perpendicular to both tangents. Before flux, write the required orientation and test the normal at one easy point. Split a closed surface into pieces, each with its own outward normal.*

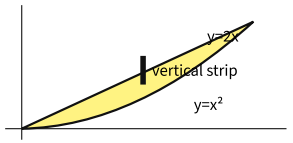
8 · Double Integrals

CH 6–7 · SETUP FIRST

Sketch before limits. Type I: a≤x≤b, $g_1(x)\le y\le g_2(x)$ (vertical strips). Type II: c≤y≤d, $h_1(y)\le x\le h_2(y)$ (horizontal strips). Choose the order giving one region description or an elementary inner antiderivative.

FUBINI + BOUNDS

```
∬_D f dA = iterated integral in either valid order
m≤f≤M ⇒ m·Area(D)≤∬_D f dA≤M·Area(D)
```



Type I region: lower curve to upper curve at fixed x. Source figure: Ch 6.

POLAR COORDINATES

```
x=r cosθ, y=r sinθ, x²+y²=r², dA=r dr dθ
Polar rectangle a≤r≤b, α≤θ≤β has area ½(b²-a²)(β-α).
```

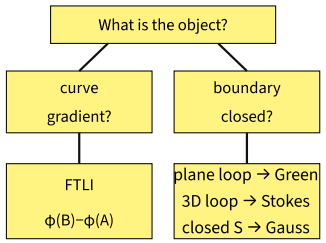
APPLICATION	INTEGRAL
Mass	$m=\iint_D \rho\,dA$
Centre of mass	$\bar{x}=(1/m)\iint_D x\rho\,dA$; $\bar{y}=(1/m)\iint_D y\rho\,dA$
Joint density	ρ≥0; total integral=1; probability=integral over event region

TRAP *Convert the region, integrand and area element on separate lines. Missing r is the classic failure.*

Application audit: density multiplies the element; moments insert x or y before division by mass. Use symmetry before integration, but state which reflection leaves both the region and density unchanged. A joint density must be non-negative and integrate to one on its full support.

12 · Theorem Chooser

PRACTICE 19 · ROUTE



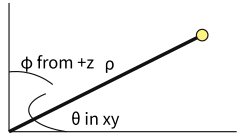
Choose the theorem from the geometric object before computing. Source figure: practice 19.

9 · Triple Integrals & Jacobians

CH 8–9 · COORDINATE CHOICE

SYSTEM	ELEMENT	NATURAL BOUNDARIES
Cartesian	dx dy dz	Planes, boxes
Cylindrical	r dr dθ dz	Cylinders, z=c±r², z=r
Spherical	ρ²sinϕ dρ dϕ dθ	ρ=c; cone ϕ=c

Convention: ρ is distance from origin; ϕ is measured down from +z; θ is the xy-plane angle. State it before limits. A type I solid has u(x,y)≤z≤v(x,y) above D; integrate z first, leaving a double integral.



Spherical convention used in the course. Source figure: Ch 8.

CHANGE OF VARIABLES

```
∬_R f(x,y)dA = ∬_S f(x(u,v),y(u,v)) |∂(x,y)/∂(u,v)| du dv
```

The absolute Jacobian is the local area scale. Choose u,v from repeated boundary/integrand combinations; map every boundary, determine the new region, substitute f and multiply by |J|.

12b · Theorem Equations

CH 10–12 · CONDITIONS

TOOL	EQUATION + CONDITION
FTLI	$F=\nabla\phi \Rightarrow \int_C F\cdot dr=\phi(B)-\phi(A)$.
Green	$\oint_C P\,dx+Q\,dy=\iint_D (Q_x-P_y)dA$; positive plane boundary.
Stokes	$\oint_C F\cdot dr=\iint_S (\nabla\times F)\cdot dS$; right-hand matched orientations.
Divergence	$\iint_S F\cdot dS=\iiint_E \nabla\cdot F\,dV$; closed surface, outward.

```
∇×F=(R_y-Q_z, P_z-R_x, Q_x-P_y)
∇·F=P_x+Q_y+R_z
```

Conservative plane field: $P_y=Q_x$ plus a domain with no holes. Build ϕ by integrating P in x, add h(y), differentiate and match Q.

ORIENTATION *Green keeps the region left; Stokes uses the right-hand rule; Gauss means outward. A hole reverses the inner boundary.*

HYPOTHESES BEFORE EQUALITY

For a conservative-field shortcut, state the no-holes domain condition as well as $P_y=Q_x$. For Stokes, the replacement surface must have the same boundary and compatible orientation. For Gauss, close an open surface with a simple lid, compute total outward flux, then subtract the lid contribution with its outward normal.

MTH2010 Worked Drills · Traps · Exam-Morning Plan

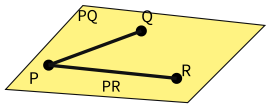
MONASH UNIVERSITY · LEVEL 2 · SEMESTER 2, 2026 · EXAM REVISION SHEET · SIDE 2 OF 2

Full guide + ask Sia about MTH2010

SIDE 2/2 · FIVE WORKED DRILLS · IF-X-THEN-Y · ERROR CHECKS · SELF-TEST BEFORE LOOKING · v2 · 2026-10-25

13 · Fast Geometry Drills

CH 1–3 · SELF-TEST



Three points → two edges → cross product → plane. Source figure: Ch 1.

30-SECOND RECALL

IF YOU SEE	DO THIS FIRST
Plane through 3 points	Form both edges from the same point.
Curve tangent	Solve $r(t_0)=p$ before $r'(t_0)$.
Curve intersection	Use separate parameters; evaluate both tangents at the meeting point.
Surface tangent	Identify normal: ∇F or $r_u \times r_v$.

MICRO-CHECK: PLANE THROUGH P=(1,2,0), Q=(3,2,1), R=(1,4,2)

$PQ=(2,0,1)$, $PR=(0,2,2)$; $PQ\times PR=(-2,-4,4) \parallel (1,2,-2)$.

$x+2y-2z=5$. Q: $3+4-2=5$; R: $1+8-4=5$.

TRAP *A sign error in the cross product is exposed by substituting all original points.*

14 · Limit / Approximation Cards

CH 2-3 · JUDGEMENT

If two paths disagree → limit does not exist. **If all lines agree →** no conclusion; try a curve or find a uniform bound. **If polar leaves θ →** bound it independently of θ.

LINEARISATION CHECK: F=XE^Y AT (2,0)

$f=2$, $f_x=1$, $f_Y=2$ ⇒ $L=2+(x-2)+2y$.
$L(2.1,-0.05)=2.0$; true value ≈ 1.998.
Continuous partials justify the estimate.

If partials merely exist → do not claim differentiability. **If a function is discontinuous →** it cannot be differentiable (contrapositive).

CLAIM TO MAKE	EVIDENCE REQUIRED
Limit fails	Two explicit paths with different outputs.
Limit exists	Limit laws or a bound valid in every direction.
Continuous extension	Existing finite limit, then define the missing value to equal it.
Differentiable	Definition, or continuous first partials nearby.

For level curves f=c, list all c-cases: ordinary curves, a possible single point at an extreme level, and the empty set. The non-empty cases give the range.

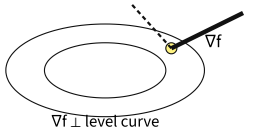
15 · Worked 1: Directional Rate

CH 4 · ORIGINAL VALUES

For $f=x^2y+yz$ at (1,2,−1), find the rate along v=(2,1,2) and the maximum rate.

1. $\nabla f=(2xy, x^2+z, y)=(4,0,2)$.
2. $|v|=3$, so $u=v/3$.
3. $D_u f=(4,0,2)\cdot(2,1,2)/3=12/3=\mathbf{4}$.
4. Maximum rate $|\nabla f|=\sqrt{20}=\mathbf{2\sqrt{5}\approx 4.47}$, along (4,0,2).

FAILURE MODE *Using v rather than u returns 12, three times the directional derivative.*



At a level curve the gradient gives the steepest direction and the normal. Source figure: Ch 4.

16 · Worked 2: Critical Points

CH 5 · ORIGINAL VALUES

$f=x^3-3x+y^2-4y$. Set $f_x=3x^2-3=0$ and $f_Y=2y-4=0$ ⇒ candidates **(1,2), (−1,2)**.

$f_{xx}=6x$, $f_{xy}=0$, $f_{yy}=2$, so $D=12x$.

POINT	D,A	DECISION + VALUE
(1,2)	12, 6	Local min; f=−6
(−1,2)	−12, −6	Saddle; f=−2

Answer: local minimum −6 at (1,2); saddle at (−1,2); no local maximum.

27 · Exam-Morning Plan

COURSE STUDY STRATEGY

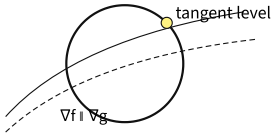
1. Write from memory: $D=AC-B^2$; all coordinate elements; both line-integral formulas; Green/Stokes/Gauss.
2. Draw the object or region before algebra.
3. Name the theorem; verify hypotheses and orientation.
4. Keep candidates, bounds and evaluation points visible.
5. Recompute one numerical anchor; run sign, scale and substitution checks.

Self-test rule: cover the answer, say the first line aloud, then write it. If the first line is wrong, classify the error as definition, region, coordinate element, hypothesis or orientation and repeat that family.

PROMPT	FIRST LINE TO RECITE
Composite at a point	Find the inner point; list every dependency route.
Implicit derivative	Check point, continuity and non-zero dependent partial.
Directional rate	Compute ∇f; normalise the direction.

17 · Optimisation Judgement

CH 5 · CANDIDATE DISCIPLINE



Constrained extremum: a level curve touches the constraint. Source figure: Ch 5.

IF YOU SEE	DO
Closed bounded region	Interior + every edge + corners; compare values.
Smooth constraint g=0	$\nabla f=\lambda\nabla g$ plus constraint; check zero cases before division.
Unbounded constraint	Find candidates and inspect end behaviour.
D=0	Inspect f along lines/curves or complete the square.

LAGRANGE CHECK: F=X+Y, X²+Y²=2

$(1,1)=\lambda(2x,2y)$ ⇒ $x=y$; $2x^2=2$ ⇒ $(\pm1,\pm1)$ with matching signs. Values 2 and −2. Circle closed and bounded ⇒ absolute max/min.

18 · Worked 3: Choose the Order

CH 6 · ORIGINAL VALUES

Evaluate $\iint_D x\,dA$ where D is bounded by $x=y^2$ and $x=4$.

1. Intersections $y^2=4$ ⇒ $-2\le y\le 2$.
2. Horizontal strip: $y^2\le x\le 4$; no split.
3. $\int_{-2}^{2}\int_{y^2}^{4} x\,dx\,dy=\tfrac12\int_{-2}^{2}(16-y^4)dy$.
4. $\tfrac12[16y-y^5/5]_{-2}^{2}=\tfrac12(64-64/5)=\mathbf{128/5=25.6}$.

CHECK *A vertical-strip setup needs a split above and below the x-axis; the horizontal-strip setup does not.*

26 · Classic Traps

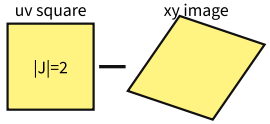
ALL CHAPTERS · FINAL SCAN

WRONG MOVE	REPAIR
Claim limit from paths	Paths disprove; prove with a global bound.
Use v in $D_v f$	Normalise v first.
Forget B^2 in D	Write $D=AC-B^2$ first.
Ignore corners	Add every non-smooth boundary point.
Drop r or $\rho^2\sin\phi$	Write the element separately.
Use $P_Y=Q_X$ across a hole	Check the domain.
Wrong theorem sign	Draw arrows/normals first.

Candidate ledger: label every value "interior", "edge", "corner" or "constraint". A list of stationary points is not an absolute-extrema answer until all boundary candidates are included and their function values are compared.

19 · Multiple-Integral Cards

CH 6-9 · SETUP



x=u+v, y=u−v maps a unit square to a tilted square of area 2. Source figure: Ch 9.

IF YOU SEE	CHOOSE / WRITE FIRST
Disc/annulus; x^2+y^2	Polar; dA=r dr dθ.
Axisymmetric solid in r,z	Cylindrical; dV=r dr dθ dz.
Sphere at origin / cone	Spherical; $dV=\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$.
Repeated linear combinations	Let them be u,v; map boundaries and compute |J|.
Awkward inner antiderivative	Redraw; reverse order before integrating.

COORDINATE ELEMENT AUDIT

- Sketch the region or a cross-section.
- Write bounds, transformed integrand and element separately.
- Ensure the inner variable's bounds may depend only on outer variables.
- Use symmetry only after checking the region and integrand share it.
- For volume, compare with a simple enclosing solid.

RADIAL DISC CHECK

$\iint_{x^2+y^2\le4} e^{-(x^2+y^2)}dA$
$=2\pi\int_0^2 r\,e^{-r^2}dr=\pi(1-e^{-4})\approx3.084$.

20 · Spherical Self-Test

CH 8 · VALUES

For $\iiint_{ball} z^2 dV$ over $x^2+y^2+z^2\le1$: $z=\rho\cos\phi$; multiply by $\rho^2\sin\phi$. The integrand is $\rho^4\cos^2\phi\sin\phi$; bounds $\rho:0\to1$, $\phi:0\to\pi$, $\theta:0\to2\pi$.

$(\int_0^1\rho^4 d\rho)(\int_0^{\pi}\cos^2\varphi\,\sin\varphi\,d\varphi)(\int_0^{2\pi}d\theta)$
$=(1/5)(2/3)(2\pi)=4\pi/15$.

BOUND SURFACE	COORDINATE EQUATION
Vertical cylinder radius a	r=a
Sphere radius a	ρ=a
Cone about +z	ϕ=constant
Horizontal plane	z=constant

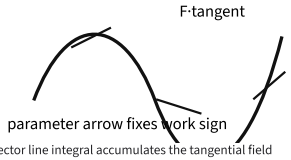
Read angles from the sketch: θ sweeps the xy-projection; ϕ sweeps down from +z. Test one boundary point against every inequality before integrating.

BOUNDS CHECK *Sketch a cross-section to find the inner variable's surfaces and angle ranges. Write the element before substitution; test a boundary point against each inequality and compare with an enclosing solid.*

21 · Line / Surface Integral Split

CH 9–11 · DO NOT MIX

QUESTION SIGNAL	REQUIRED OBJECT	DIRECTION EFFECT
Wire mass / arc length	$\|r'(t)\|dt$	None
Work / circulation	$F(r(t))\cdot r'(t)dt$	Reverse ⇒ negate
Surface area / shell mass	$\|r_u\times r_v\|du\,dv$	None
Flux	$F\cdot(r_u\times r_v)du\,dv$	Reverse normal ⇒ negate



Vector line integral accumulates the tangential field component along the oriented curve. Source figure: Ch 9.

HELIX LENGTH MICRO-DRILL

$r=(3\cos t, 3\sin t, 4t)$, $0\le t\le 2\pi$.
$r'=(-3\sin t, 3\cos t, 4)$, $|r'|=\sqrt{25}=5$ ⇒
$L=\int_0^{2\pi}5dt=\mathbf{10\pi}$.

TRAP *A scalar line integral uses a speed; a work integral uses a dot product. A flux integral uses a chosen normal, not a magnitude.*

22 · Surface Area Check

CH 11 · ORIGINAL VALUES

Plane $2x+2y+z=4$ in the first octant: $z=4-2x-2y$ over triangle D: $x,y\ge0$, $x+y\le2$, $A(D)=2$.

$dS=\sqrt{1+(-2)^2+(-2)^2}dA=3dA$
$A(surface)=3\cdot A(D)=\mathbf{6}$.

For a plane, the surface factor is constant. Compute the projected area before setting up an unnecessary integral.

24 · Vector-Theorem Cards

CH 10-12 · CHEAPEST ROUTE

IF YOU SEE	DO Y
Open path; $P_Y=Q_x$; no holes	Recover a potential; subtract endpoints.
Closed plane curve	Consider Green.
Space curve bounding a surface	Use Stokes; choose an easier surface.
Closed surface	Consider divergence theorem.
Open flux surface	Add a lid; total outward flux − lid flux.

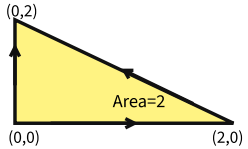
Potential: $\phi=\int P\,dx+h(y)$; differentiate in y; match Q; verify $\nabla\phi=F$. **Area by Green:** $P=-y/2$, $Q=x/2$ gives $Q_x-P_Y=1$.

ORIENTATION PHRASE	IMMEDIATE ACTION
Anticlockwise plane boundary	Positive for Green.

23 · Worked 4: Green

CH 10 · ORIGINAL VALUES

Evaluate $\oint(y\,dx+3x\,dy)$ anticlockwise around the triangle (0,0),(2,0),(0,2).



Anticlockwise boundary keeps the triangular region on the left. Source figure: Ch 10.

1. $P=y$, $Q=3x$ ⇒ $Q_x-P_Y=3-1=\mathbf{2}$.
2. Orientation is positive; triangle area $=\tfrac12\cdot2\cdot2=\mathbf{2}$.
3. Green: $\oint P dx+Q dy=\iint_D 2\,dA=2\cdot2=\mathbf{4}$.

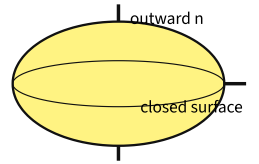
SIGN CHECK *Clockwise travel would give −4.*

25 · Worked 5: Divergence

CH 12 · ORIGINAL VALUES

For $F=(x^3,y^3,z^3)$ through the outward unit sphere: $\nabla\cdot F=3\rho^2$. The surface is closed, so

$flux=3\int_0^{2\pi}\int_0^{\pi}\int_0^1\rho^4\sin\varphi\,d\rho\,d\varphi\,d\theta$
$=3\cdot(1/5)\cdot4\pi=\mathbf{12\pi/5\approx7.54}$.



Closed surface + outward flux points to the divergence theorem. Source figure: Ch 12.

TRAP *Convert the divergence, not the vector field, to spherical coordinates.*

FULL-MARK THEOREM LINE

Write: theorem name → why it applies → orientation → transformed interior integral → limits → value. If replacing a Stokes surface, state "same boundary, compatible orientation". If adding a Gauss lid, show wanted flux = closed flux − lid flux before numbers.

Final plausibility: compare sign with field direction, magnitude with surface/volume scale, and units with the quantity (length, area, volume, work or flux).

LAST 20-SECOND CHECK	QUESTION
Geometry	Does the normal/orientation match the words?
Calculus	Were all factors and bounds carried through?
Theorem	Were its hypotheses actually satisfied?
Answer	Is the conclusion named, signed and simplified?