MTH1020 Analysis of Change

MONASH UNIVERSITY · EXAM REVISION SHEET · SEMESTER 2, 2026

Full guide + ask Sia about MTH1020

SIDE 1 · RECALL & CHECK

Priority: definition → method choice → formula → assumption → check

CALCULUS CORE + FOUNDATIONS

0 · Exam Map

COURSE.JSON · BRIEF

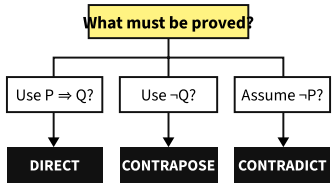
Q 15 | MST 30 | PSA 25 | Final 30

ASSESSMENT	WEIGHT
Applied-class quizzes	15%
Mid-semester tests	30% (all three combined)
Problem-solving activities / online assignments	25%
Final examination	30%

Permitted materials: not confirmed for this offering in the course materials we hold — check the exam instructions on the LMS. *[BRIEF; course facts only]*

1 · Proof Chooser

CH 1



False ∀-claim? Give one counterexample.

Name the method in line 1; finish with the claimed conclusion.

SIGNAL	MOVE
∀x P(x)	general proof; disprove with one witness
∃x P(x)	exhibit a witness; disprove every case
P ⇒ Q	direct or prove ¬Q ⇒ ¬P
Irrationality	assume a/b in lowest terms; force common factor

NEGATION BANK

```
¬(∀x P) ≡ ∃x ¬P · ¬(∃x P) ≡ ∀x ¬P
¬(P ∧ Q) ≡ ¬P ∨ ¬Q · ¬(P ∨ Q) ≡ ¬P ∧ ¬Q
```

Inequality engine: move all terms to one side and recognise a square. $x^2 \geq 0$; $a^2+b^2 \geq 2ab$; for $x,y \geq 0$, $(x+y)/2 \geq \sqrt{(xy)}$, equality iff x=y.

DEFINITIONS THAT UNLOCK THE ALGEBRA

```
even n ⇔ n=2k · odd n ⇔ n=2k+1 · m divides n ⇔ n=km for some k∈ℤ
ℕ⊂ℤ⊂ℚ⊂ℝ⊂ℂ · {x∈A:R(x)} = elements of A satisfying R.
```

Converse trap: P⇒Q says nothing by itself about Q⇒P. Only the contrapositive ¬Q⇒¬P is equivalent. When negating x>5, keep the boundary: x≤5, not x<5.

FULL-MARK PROOF LINE	WHAT IT DOES
"Suppose…" / "Assume…"	declares the permitted starting facts
"Therefore…" with reason	makes every implication checkable
"This contradicts…"	names the exact incompatible facts

2 · Induction & Functions

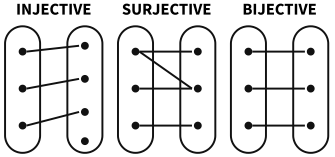
CH 2

INDUCTION OBLIGATION

```
Base: prove P(1).
Hypothesis: assume P(k) for one arbitrary k∈ℕ.
Step: use it to prove P(k+1).
Conclude P(n) for every n∈ℕ.
```

Divisibility step: rewrite the (k+1)-case until the induction-hypothesis multiple appears; show the remainder is also divisible. **Sum step:** start with the sum to k+1, split off the final term, then substitute P(k).

```
1/[j(j+1)] = 1/j - 1/(j+1)
Σ_{j=1}^{n} 1/[j(j+1)] = 1 - 1/(n+1) = n/(n+1)
```



Horizontal-line logic: at most once · at least once · exactly once.

PROPERTY	DEFINITION / CHECK
Injective	f(a)=f(b) ⇒ a=b; horizontal line ≤1
Surjective	range = co-domain; every target hit
Bijective	both; only then f^{-1} exists
Even / odd	f(−x)=f(x) / f(−x)=−f(x)
Periodic	f(x+T)=f(x)

```
(g∘f)(x)=g(f(x)); need range f ⊆ domain g.
f⁻¹(f(x))=x; f⁻¹(x) ≠ 1/f(x).
```

FUNCTION WORKFLOW

Natural domain: exclude zero denominators; require even-root radicands ≥0; require log arguments >0. State the set before classifying the function. **Range** means outputs actually achieved; it can be smaller than the co-domain.

Inverse: first verify/restrict to a bijection; write y=f(x), solve for x, then swap labels. The graphs of f and f^{-1} reflect in y=x. Verify both compositions on the correct domains.

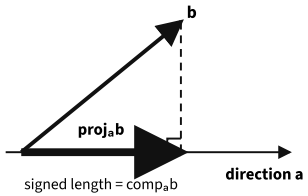
CANONICAL TARGET	STEP ENGINEERING
$\Sigma_{i=1}^{n}(2i-1)=n^2$	add 2k+1 → $k^2+2k+1=(k+1)^2$
n^3-n divisible by 3	expand $(k+1)^3-(k+1)$; expose k^3-k
11^n-1 divisible by 10	write $11(11^k-1)+10$
telescoping	expand first/last terms; show cancellations

3 · Vectors: Dot → Cross

CH 3–4

DOT-PRODUCT FAMILY

```
|v⃗|=√(v₁²+v₂²+v₃²) · v̂=v/|v|
a·b=Σaᵢbᵢ=|a||b|cosθ
cosθ=(a·b)/(|a||b|) · a⊥b ⇔ a·b=0
comp_a b=(a·b)/|a| · proj_a b=[(a·b)/|a|²]a
```



Scalar projection is a number; vector projection is the arrow along a.

Denominator check: scalar ÷|a|; vector ÷|a|². Sign check: a·b<0 gives an obtuse angle and a negative scalar projection.

CROSS PRODUCT

```
a×b=(a₂b₃−a₃b₂, a₃b₁−a₁b₃, a₁b₂−a₂b₁)
|a×b⊨|a||b|sinθ · a×b=-(b×a)
parallelogram area=|a×b|; triangle area=½|AB×AC|
```

NEED	USE	OUTPUT
angle, projection, ⊥ test	dot / cos	scalar
normal, area	cross / sin	vector
unit direction	divide by magnitude	vector of length 1

COMPONENT DISCIPLINE

```
AB = position(B)-position(A)
(a₁,a₂,a₃)±(b₁,b₂,b₃)=(a₁±b₁,a₂±b₂,a₃±b₃)
v·v=|v|² · work=force·displacement
```

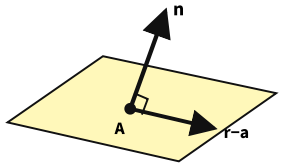
Angle range: arccos returns 0≤θ≤π. Positive dot ⇒ acute; zero ⇒ right; negative ⇒ obtuse. A zero cross product means parallel inputs (or a zero vector); swapping the input order reverses the normal.

ANSWER-SHAPE CHECK	EXPECTED
magnitude / distance / scalar projection	one real number; distance non-negative
vector projection / normal	ordered components
angle	arccos expression or stated degree/radian value
area	non-negative; triangle is half the cross magnitude

CHECK → *Dot your computed cross product with both inputs: both answers must be 0.*

4 · Lines, Planes & Complex

CH 4–6



(r−a)·n=0 ⇔ r·n=a·n

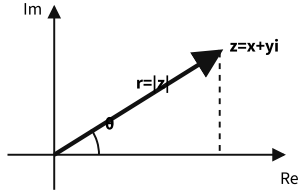
A point plus a normal fixes a plane; coefficients are the normal components.

```
Line: r(t)=a+td. Through A,B: d=B-A.
Plane: (r-a)·n=0 ⇔ n₁x+n₂y+n₃z=a·n.
Through A,B,C: n=AB×AC.
```

QUESTION	RECIPE
Line intersection	test $d_1=kd_2$; solve $r_1(t)=r_2(u)$
Point–plane distance	$\lvert PQ\cdot\hat{n}\rvert$
Skew-line distance	$\lvert(a_2-a_1)\cdot\hat{n}\rvert$, $\hat{n}=(d_1\times d_2)/\lvert d_1\times d_2\rvert$
Angle of planes	angle between normals

COMPLEX CHECKPOINT · CONFIRM FINAL SCOPE

The held materials say complex numbers were excluded from the S1-2026 final but tested in mid-semester tests; confirm your cohort's scope on Moodle.



Read the quadrant before choosing θ; arctan(y/x) alone is unsafe.

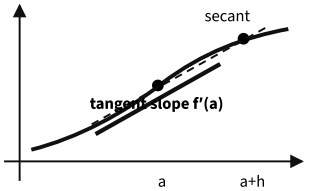
```
z̄=x-yi · z z̄=x²+y²=|z|² · z₁/z₂=z₁z̄₂/|z₂|²
z=r cisθ=r e^{iθ} · z^n=r^n cis(nθ)
n-th roots: r^{1/n} cis[(Arg z+2πk)/n], k=0,…,n-1.
```

COMPLEX CUE	MOVE / CHECK
divide by c+di	multiply top and bottom by c−di; denominator c^2+d^2
power i^n	reduce n modulo 4: 1,i,−1,−i
power of x+yi	convert to r cisθ, then De Moivre
n-th roots	k=0,…,n−1; equal spacing 2π/n
real-coefficient polynomial	non-real roots occur in conjugate pairs

```
|z-z₀|⊨ρ: circle centred z₀ · Re(z)=c: vertical line · Arg(z)=α: ray.
```

5 · Differentiate Correctly

CH 7



Derivative = limiting secant slope as h→0.

```
f′(a)=lim_{h→0} [f(a+h)-f(a)]/h
continuous at a ⇔ lim_{x→a} f(x)=f(a)
differentiable ⇒ continuous; converse false.
```

STRUCTURE	RULE
f·g	(fg)′=f′g+fg′
f/g	(f/g)′=(f′g−fg′)/g^2
f(g(x))	f′(g(x))g′(x)
relation in x,y	differentiate y as y(x); collect dy/dx
products / x^k	take ln first; differentiate implicitly

STANDARD DERIVATIVES

```
(xⁿ)′=nx^{n-1} · (eˣ)′=eˣ · (ln x)′=1/x
(sin x)′=cos x · (cos x)′=-sin x · (tan x)′=sec²x
(Sin⁻¹x)′=1/√(1-x²) · (Cos⁻¹x)′=-1/√(1-x²)
(Tan⁻¹x)′=1/(1+x²)
```

FAILURE	WHY F′ FAILS
|x| at 0	left slope −1; right slope +1
jump	not continuous, hence not differentiable
√x at 0	vertical tangent; slope undefined

RULE-RECOGNITION CHECKPOINT

Composition? Differentiate the outside, keep the inside, multiply by the inner derivative. **Product?** both terms survive: f′g+fg′. **Quotient?** preserve the order f′g−fg′ and square the denominator.

```
Tangent at x=a: y-f(a)=f′(a)(x-a).
Normal slope = -1/f′(a) when f′(a)≠0.
Two-sided limit exists only when left and right limits agree.
```

Log differentiation: for $y=x^x$, ln y=x ln x, so y′/y=ln x+1 and $y'=x^x(\ln x+1)$. For a product, logs turn multiplication into addition before differentiating.

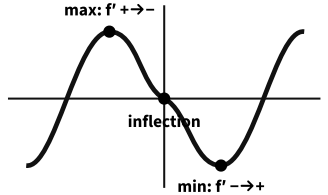
CURVE-SKETCH ORDER

1. Domain and intercepts.
2. Vertical/horizontal asymptotes and end behaviour.
3. Critical points; sign chart for f′.
4. Possible inflections; sign chart for f″.
5. Plot labelled points and join consistently.

IVT root test: if f is continuous on [a,b] and f(a),f(b) have opposite signs, some c∈(a,b) has f(c)=0. This proves existence, not uniqueness or an exact value.

6 · Shape, Models & Integrals

CH 8–12



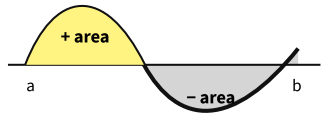
Stationary points need classification; f″=0 alone does not prove inflection.

TEST	CONCLUSION
f′: +→− / −→+	local max / min
f″(c)<0 / >0, f′(c)=0	max / min
f″(c)=0	inconclusive; test sign
continuous on [a,b]	absolute max/min attained; compare critical points + endpoints

Related rates: draw → name changing quantities → write relation → differentiate with respect to t → substitute the instant last → solve with units.

Optimisation: objective + constraint → eliminate to one variable → state domain → solve f′=0 → justify absolute extremum → answer in words. Growth as x→∞: logarithms ≪ powers ≪ exponentials.

MODEL CUE	NON-NEGOTIABLE
rate "at the instant"	differentiate first, substitute values last
maximum/minimum on interval	include endpoints
finite limit at infinity	horizontal asymptote y=L
denominator→0	test signs from both sides



$\int_a^b f = F(b)-F(a)$, where F′=f

A definite integral is signed area; geometric area stays non-negative.

FTC + ANTIDERIVATIVES

```
∫_a^b f(x)dx=F(b)-F(a) · d/dx ∫_a^x f(t)dt=f(x)
∫xⁿdx=x^{n+1}/(n+1)+C (n≠-1)
∫dx/x=ln|x|+C · ∫eˣdx=eˣ+C · ∫cosx dx=sinx+C
```

TECHNIQUE CHOOSER

SEE	DO
f(g(x))g′(x)	u=g(x); convert limits if definite
unlike product	parts: ∫fg′=fg−∫f′g
ratio of polynomials	divide if improper; partial fractions

Forward this to your tutorial group

V2 · 2026-10-25 · SHARED BY _____

Monash Semester 2 final assessment period, 2–18 November 2026 (check your personal exam timetable). **Check your exam instructions on the LMS for what you may bring; use this sheet to revise and self-test.**

MTH1020 Analysis of Change · Five Self-Test Drills

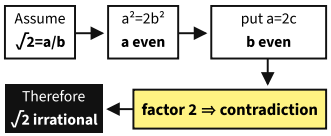
MONASH UNIVERSITY · ANSWER PATHS, JUDGEMENT CARDS & CLASSIC TRAPS

Full guide + ask Sia about MTH1020

SIDE 2 · DRILL & DIAGNOSE — **Method:** hide the steps · write the route · compute · name the check — **5 WORKED MINI-EXAMPLES**

Drill 1 · Contradiction CH 1 WORKED

Prompt: Prove $\sqrt{2}$ is irrational. [4 marks]



Justify every arrow; use lowest terms.

The assumption supplies an equation; parity forces the forbidden common factor.

1. **Set-up (+1):** suppose $\sqrt{2}=a/b$, $a,b\in\mathbb{Z}$, $b\neq0$, and a/b is in lowest terms.
2. **First parity (+1):** $a^2=2b^2$, so a^2 is even. Since an odd square is odd, a is even; write a=2c.
3. **Second parity (+1):** $(2c)^2=2b^2 \Rightarrow b^2=2c^2$, so b is even.
4. **Close (+1):** both share factor 2, contradicting lowest terms. Hence $\sqrt{2}$ is irrational. □

IF YOU SEE...	DO...
"for every n∈ℕ"	base + hypothesis + step
universal claim false?	give one valid counterexample
n^2 even ⇒ n even	prove contrapositive: odd ⇒ odd square

INDUCTION MEMORY TEST

Target: $1+3+\cdots+(2n-1)=n^2$. Without expanding every term, say the four lines: P(1): $1=1^2$; assume the sum to k equals k^2; add next term $2k+1$; $k^2+2k+1=(k+1)^2$; conclude by induction.

Divisibility version: the induction hypothesis must be written as an integer multiple, e.g. $11^k-1=10m$ for some $m\in\mathbb{Z}$. Then $11^{k+1}-1=11(11^k-1)+10=10(11m+1)$.

COMMUNICATION CHECK	REQUIRED WORDING
base	substitute the first permitted n and compare both sides
hypothesis	"Assume P(k) for some k∈ℕ"
step	show exactly where P(k) is used
finish	invoke the principle and quantify all n

TRAP → *Examples support a guess; they never prove a universal statement.*

PARITY SUB-LEMMA

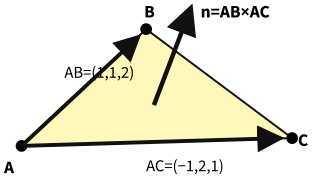
Needed twice above: if n^2 is even, then n is even.

Prove the contrapositive: if n is odd, $n=2k+1$, so $n^2=4k^2+4k+1=2(2k^2+2k)+1$ is odd. Thus n^2 even forces n even.

LOWEST-TERMS CHECK → *"a and b are even" clashes with the starting condition that a/b has no common factor greater than 1; name that exact contradiction.*

Drill 2 · Plane from 3 Points CH 4 WORKED

Prompt: A(1,0,1), B(2,1,3), C(0,2,2). Find AB×AC, triangle area and the plane. [4 marks]



Two edges give a normal; its magnitude gives the parallelogram area.

1. **Edges (+1):** AB=B−A=(1,1,2); AC=C−A=(−1,2,1).
2. **Cross (+1):** AB×AC=(1·1−2·2, 2·(−1)−1·1, 1·2−1·(−1))=(−3,−3,3).
3. **Area (+1):** $|AB\times AC|=\sqrt{27}=3\sqrt{3}$, so triangle area=$3\sqrt{3}/2\approx2.60$.
4. **Plane (+1):** scale normal to (1,1,−1). Through A: x+y−z=1+0−1=0.

ANSWER + CHECK

AB×AC=(-3,-3,3) · area=3√3/2 · plane x+y-z=0.
B: 2+1-3=0 ✓ · C: 0+2-2=0 ✓

CLASSIC SLIP	REPAIR
AC=C−B	form both edges from the same point A
triangle area=|cross|	halve the parallelogram area
wrong cross sign	dot result with AB and AC; both 0
plane constant guessed	compute k=a·n; check spare points

DOT-PRODUCT MICRO-DRILL

For a=(2,−1,2), b=(1,3,−1): a·b=−3, |a|=3, |b|=$\sqrt{11}$. Therefore $\theta=\arccos(-1/\sqrt{11})\approx107.5°$, $\text{comp}_a b=-1$, and $\text{proj}_a b=(-3/9)a=(-2/3,1/3,-2/3)$.

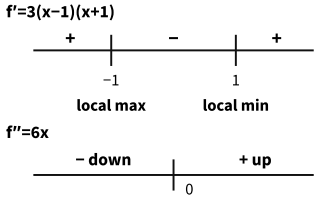
Sign triangle: a·b<0 ⇔ θ obtuse ⇔ scalar projection negative.
Projection denominator: |a| for the signed length; |a|² for the vector.

3-D GEOMETRY QUESTION	FIRST CONSTRUCTION
plane through 3 points	two same-tail edges, then cross
are lines parallel?	compare direction vectors for scalar multiples
do lines meet?	solve all components of $r_1(t)=r_2(u)$
skew distance	unit common normal from $d_1\times d_2$

SHAPE CHECK → *An angle is a scalar; a normal and a projection are vectors; a plane is an equation.*

Drill 3 · Cubic Shape CH 8 WORKED

Prompt: For $f(x)=x^3-3x$, find and classify critical points and any inflection point. [4 marks]



The signs, not merely the zeroes, make the classifications.

1. **Critical points (+1):** $f'=3x^2-3=3(x-1)(x+1)=0 \Rightarrow x=-1,1$.
2. **Classify (+1):** $f''=6x$. $f''(-1)=-6<0 \Rightarrow$ local max; $f''(1)=6>0 \Rightarrow$ local min.
3. **Coordinates (+1):** $f(-1)=2$ and $f(1)=-2 \Rightarrow$ max $(-1,2)$, min $(1,-2)$.
4. **Inflection (+1):** $f''=0$ at x=0 and changes −→+; f(0)=0, so (0,0).

ANSWER

local max (-1,2) · local min (1,-2) · inflection (0,0)

JUDGEMENT	FAST CHECK
local extremum?	f′ changes sign; stationary alone is insufficient
inflection?	f″ changes sign; f″=0 alone is insufficient
absolute on [a,b]?	compare all interior critical values and both endpoints
asymptote?	finite limit at ±∞ → horizontal; denominator→0 → test each side

DERIVATIVE TRAPS UNDER PRESSURE

EXPRESSION	CORRECT FIRST LINE
$x^2\sin(3x)$	$2x\sin(3x)+x^2\cdot3\cos(3x)$
$x^2+y^2=25$	$2x+2y(dy/dx)=0$
sin(g(x))	cos(g(x))g′(x)
f(x)/g(x)	$[f'g-fg']/g^2$

Interpret before sketching: f′>0 means increasing, f′<0 decreasing; f″>0 concave up, f″<0 concave down. A critical point also includes where f′ does not exist, not only where f′=0.

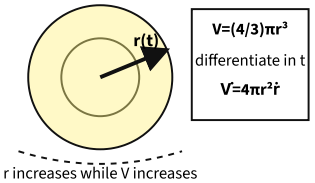
Closed-interval absolute extrema: EVT guarantees attainment for a continuous f on [a,b]. Compute f at every critical point inside (a,b) and at both endpoints; compare the actual values.

COUNTER-CHECK → *x^4 has $f''(0)=0$ but $f''=12x^2\geq0$ on both sides: no inflection.*

Second-derivative test: if f′(c)=0 and f″(c)>0, c is a local minimum; if f″(c)<0, a local maximum. If f″(c)=0, the test is inconclusive—use the first-derivative sign change.

Drill 4 · Related Rates CH 9 WORKED

Prompt: A spherical balloon receives air at dV/dt=100 cm^3/s. Find dr/dt when r=5 cm. [4 marks]



Keep r variable through differentiation; substitute r=5 only afterwards.

1. **Relate (+1):** $V=(4/3)\pi r^3$; both V and r depend on t. Given V′=100; need ṙ at r=5.
2. **Differentiate (+1):** $V'=4\pi r^2\dot{r}$ by the chain rule.
3. **Substitute (+1):** $100=4\pi(5)^2\dot{r}=100\pi\dot{r}$.
4. **Solve (+1):** $\dot{r}=1/\pi\approx0.318$ cm/s.

UNIT CHECK

(cm³/s) ÷ cm² = cm/s ✓

OPTIMISATION COUSIN

Closed cylinder with fixed V: $S=2\pi r^2+2\pi rh$, $V=\pi r^2h$. Eliminate $h=V/(\pi r^2)$:

S(r)=2πr²+2V/r, r>0
S′=4πr−2V/r²=0 ⇒ r³=V/(2π)
S″=4π+4V/r³>0 ⇒ minimum
V=2πr³ ⇒ h=V/(πr²)=2r.

Conclusion: the minimal-surface closed cylinder has height equal to diameter.

TRAP	REPAIR
insert r=5 before differentiating	differentiate the variable relation first
leave two variables in objective	use the constraint to eliminate one
stop at f′=0	justify minimum/maximum on the stated domain

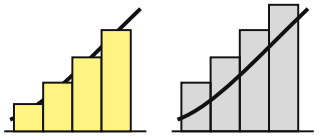
RATE AND MODEL CHECKLIST

LINE TO WRITE	WHY IT EARNS THE MARK
"V and r depend on t"	licenses the chain-rule factors
given and required derivatives	fixes signs and units before algebra
general relation before values	preserves the changing quantities
final sentence with units	answers direction and physical quantity

Sign sense: increasing quantities have positive rates; a draining height or shrinking radius has a negative rate. The rate asked for may have different units from the supplied rate, so dimensional cancellation is a compulsory check.

4b · Integration Judgement Lab CH 10–11

LEFT(4): UNDER **RIGHT(4): OVER**



For decreasing f, the inequalities reverse.

Endpoint choice controls the direction of approximation before n→∞.

$\int_a^b f(x)dx=\lim_{n\to\infty}\Sigma f(x_i^*)\Delta x$, $\Delta x=(b-a)/n$.
$\int_a^a f=0$ · $\int_a^b f=-\int_b^a f$ · $\int_a^b f=\int_a^c f+\int_c^b f$.

INTEGRAND SHAPE	FIRST MOVE	MANDATORY CHECK
$2x\cos(x^2)$	$u=x^2$; $du=2x\,dx$	$d/dx[\sin(x^2)]=2x\cos(x^2)$
$x\,e^x$	parts: f=x, g′=e^x	$d/dx[e^x(x-1)]=xe^x$
proper rational	factor denominator; partial fractions	recombine fractions
degree numerator ≥ denominator	polynomial division first	remainder is proper
$\sqrt{(a^2-x^2)}$	x=a sin u	√…=a cos u; transform dx

FAST SELF-TEST ANSWERS

$\int 2x\cos(x^2)dx=\sin(x^2)+C$
$\int x\,e^x dx=xe^x-e^x+C=e^x(x-1)+C$
$\int_0^2(3x^2-2x)dx=[x^3-x^2]_0^2=4$
$d/dx\int_1^x\cos(t^2)dt=\cos(x^2)$

Why the last derivative works: the FTC derivative form returns the integrand at the upper limit; no elementary antiderivative of $\cos(t^2)$ is required.

CLASSIC TRAP	CORRECTION
omit +C	indefinite integral is a family
keep x-limits after u-sub	convert terminals or back-substitute
integral is "area"	definite integral is signed; split for geometric area
trust answer	differentiate the antiderivative back

PARTIAL-FRACTION AND TRIG FORMS

DENOMINATOR FACTOR	TERM TO WRITE
ax+b	A/(ax+b)
$(ax+b)^2$	A/(ax+b)+B/$(ax+b)^2$
irreducible ax^2+bx+c	(Dx+E)/(ax^2+bx+c)

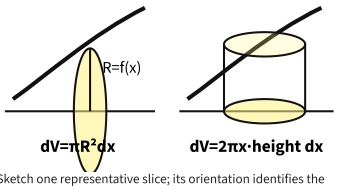
cos2x=1-2sin²x=2cos²x-1
√(a²−x²): x=a sin u, dx=a cos u du
∫dx/(x²+a²)=(1/a)Tan⁻¹(x/a)+C

By-parts choice: select f to simplify when differentiated and g′ to integrate cleanly; LIATE is a guide, not a substitute for checking whether the new integral is easier. Repeated by-parts may return the original integral, which you then solve algebraically.

Drill 5 · Volume by Discs CH 12 WORKED

Prompt: Rotate the region bounded by $y=\sqrt{x}$, the x-axis and x=4 about the x-axis. Find its volume. [4 marks]

DISCS: ⊥ TO AXIS **SHELLS: ∥ TO AXIS**



Sketch one representative slice; its orientation identifies the method.

1. **Choose (+1):** rotation about the x-axis with vertical slices ⇒ discs, $V=\pi\int_a^b[f(x)]^2dx$; a=0, b=4.
2. **Integrand (+1):** $[f(x)]^2=(\sqrt{x})^2=x$, so $V=\pi\int_0^4 x\,dx$.
3. **Integrate (+1):** $V=\pi[x^2/2]_0^4$.
4. **Evaluate (+1):** $V=\pi(16/2)=8\pi\approx25.1$ cubic units.

ANSWER

V=8π≈25.1 cubic units.

IF YOU SEE...	DO...
area between f and g	solve f=g for limits; integrate top−bottom; split if order switches
region against x-axis, rotate about x-axis	discs: π·radius²
region under f, rotate about y-axis	shells: ∫2π(radius)(height)
arc length	$\int\sqrt{(1+[f']^2)}$
surface about x-axis	$2\pi\int f\sqrt{(1+[f']^2)}$

EXAM-MORNING PLAN

1. Check your personal timetable and the LMS exam instructions; confirm your cohort's complex-number scope.
2. From memory, write the proof, induction, vector, derivative-test, related-rate, optimisation and integration chooser routes.
3. Start one problem from each family: proof, vectors, differentiation, integration. State the method before calculating.
4. Check outputs by type: scalar vs vector; coordinates not only x-values; square the disc radius; include units; differentiate antiderivatives back.
5. Write complete sentences, define variables and finish with a line answering what was asked.

FINAL SCAN → *domain · sign · endpoint · units · +C · method named · conclusion written.*

AVERAGE-VALUE RECALL

$\bar{f}=[1/(b-a)]\int_a^b f(x)dx$ · equivalently, $\int_a^b f(x)dx=(b-a)\bar{f}$.

Meaning: $\bar{f}$ is the constant height whose rectangle over [a,b] has the same signed area as the region under f.

Forward this to your tutorial group

V2 · 2026-10-25 · SHARED BY _____

Monash Semester 2 final assessment period, 2–18 November 2026 (check your personal exam timetable). **Check your exam instructions on the LMS for what you may bring; use this sheet to revise and self-test.**