EART60061 Measuring and Predicting 1

THE UNIVERSITY OF MANCHESTER · POSTGRADUATE · SEMESTER 1 2026/27

Full guide + ask Sia about EART60061

SIDE 1/2 METHODS · RULES · DECISIONS — OPEN-BOOK EXAM SHEET — Online test · 100% · v2 · 2026-10-19

0 · Exam Map

COURSE + CANVAS

ONLINE TEST 100%

"The test will be open book and will draw on course materials and the exercises/tasks you are asked to complete throughout the unit." *(EART60061 unit page, Canvas, 2026/27 1st semester)*

Format in the course data: **multiple-choice + short-answer**, covering every intended learning outcome, in **two 1.5-hour sessions**. The closest preparation is to rerun the exercises, change a parameter and explain the output.

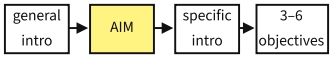
IF ASKED TO…	FIRST MOVE
quality of values	s → SE → t·SE
combine values	choose error rule
test a line	r^2 → t → p
predict process	state model + check
step a model	old values only

EXAM MOVE *Name the method, substitute with units, carry the calculation to a decision, then state the assumption or check.*

1 · Research Argument

CH 1

broad → narrow → difficulty → tasks



method organised by objectives

The written argument: the introduction earns the aim; difficulties earn ordered objectives.

QUALITY TEST

```
Research = importance × novelty;
rigour decides whether it counts.
```

Each objective must be **achievable, recognisable** and visibly linked to the aim. Order closed → open. Judge every section for coherence, completeness and concision. A Gantt chart should show parallel work and leave time for review.

PART	MARKER-FACING TEST
General introduction	broad-to-narrow case ending at the requirement
Aim	single statement, close to title
Specific introduction	why the aim is difficult; terms defined
Objectives	3–6 recognisable tasks, closed-to-open
Method	organised by objectives; replication and error explicit
Plan	parallel activities, feasible timing, review allowed

Scope honestly: an MSc project need not have global importance or novelty; state whether each is local, national or wider. Rigour appears in suitable methods, replication and quantified uncertainty.

2 · Spread: Choose the Statistic

CH 2

MEASURE	USE / WARNING
Range	quick; only min/max, hostage to one extreme
Mean absolute deviation	sign-free; no distribution assumption
Sample s	uses every value; links spread to probability
Population σ	complete population only; divisor n

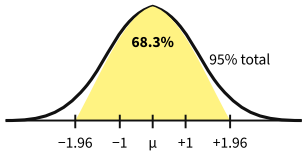
THREE SPREAD FORMULAS

```
x̄ = Σx / n
MAD = Σ|x - x̄| / n
s = √[Σ(x - x̄)²/(n-1)]
```

Why n−1? The mean was estimated from the same sample, costing one degree of freedom. Replicate field/lab data: STDEV or STDEV.S; STDEVP divides by n. Signed deviations sum to zero; squaring removes sign and weights extremes.

3 · Normal Distribution

CH 3



μ locates the peak; s sets width. 68.3% lies within ±1s and 95% within ±1.96s.

STANDARDISE

```
d = (x-x̄)/s · 95% range = x̄ ± 1.96s
within ±d: 1.00 → 68.3%; 1.96 → 95%;
2.58 → 99%
```

For a threshold, sketch first: one tail or both? Tail estimates assume random variation and that sample x̄,s represent the population. A home-built area table needs **frequency × class width**; coarse classes create resolution error.

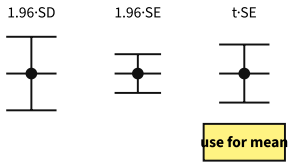
D	CENTRAL SHARE	BOTH TAILS
1.00	68.3%	31.7%
1.96	95.0%	5.0%
2.58	99.0%	1.0%

SPREADSHEET MECHANICS THAT GET TESTED

1. Set calculation to automatic before trusting changing outputs.
2. For a frequency distribution, select all output cells, enter the data range and class limits, then use the array entry method shown in the exercise.
3. Fix the mean cell with an absolute reference before copying deviation formulas.

4 · Confidence Limits

CH 4



small n widens t

Two corrections oppose: SE narrows from single readings to a mean; small-sample t widens.

FOUR-LINE METHOD

```
SE = s/√n
df = n-1
95% CL = t0.05,df × SE
report x̄ ± CL, units; interval
[x̄−CL, x̄+CL]
```

Excel: `TINV(0.05,n-1)`. Examples from the guide: df=2 → t=4.303; df=9 → 2.262; t approaches 1.96 only as n becomes very large. For two means, non-overlap supports a difference at 95%; overlap means do not claim a difference.

ERROR BAR BELONGS TO	QUANTITY
single readings	s or d·s
mean of n	SE=s/√n
small-n 95% mean	t·SE

Quote mean and limit to the same decimal places. Figure legend must state **95% confidence**. In sampling design, control tide, location, species/size and handling; replication measures variation that remains.

4b · Decision Trap

CH 4 FAQ

SD describes measurements; SE describes their mean. Using 1.96·SD for a mean is too wide; using 1.96·SE with three replicates is too narrow because t=4.303.

EXAM LINE *"At 95% confidence the intervals [do/do not] overlap; therefore a difference [cannot/can] be claimed."*

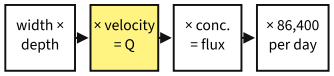
N	DF	95% T	USE
3	2	4.303	triplicate mean
4	3	3.182	four replicates
6	5	2.571	six replicates
10	9	2.262	ten replicates
∞	∞	1.960	normal limit

BIOMONITORING DESIGN → DEFENSIBLE BARS

Seaweed integrates exposure and concentrates metals, avoiding seawater's tide/current variation and salt matrix effects. Hold constant tide stage, sampling position, species and specimen size; replicate within each site. If a substitute species is unavoidable, sample both at a shared site and disclose the correction-factor ratio.

5 · Error Propagation

CH 5



exact multiplier keeps fractional error

Carry units first, then fractional errors through every measured multiplication.

SELECT BY OPERATION

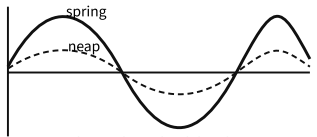
```
sum/difference: E = √(Eα²+Eβ²+…)
product/quotient: E_Q/Q = √[(Eα/A)²+
(Eβ/B)²+…]
power x^n: E/x^n = |n|·Ex/x
```

Assumptions: errors are random and uncorrelated. Direct addition overstates error at the stated confidence. An exact multiplier has zero error: it scales absolute error, not fractional error. The largest fractional input error usually dominates—identify it to say which measurement should improve.

UNIT CHAIN	RESULT
$m^3/s \times mg/L$	g/s (1000 L/m^3; 1000 mg/g)
g/s × 86,400 s/day	g/day
g/day ÷ 1000	kg/day

5b · Flux & Sampling

CH 5 CASE



hours through tidal cycle

Flux can reverse with the tide; sample complete spring and neap cycles, not isolated concentrations.

Concentration is mass/volume; **flux** is mass/time and predicts transport. Estuary flux also varies through dilution, pH, adsorption, flocculation, settling, storms and stored metal. A negative tidal-cycle flux can mean net landward movement, not an arithmetic error. Compare a consent with the whole uncertainty interval.

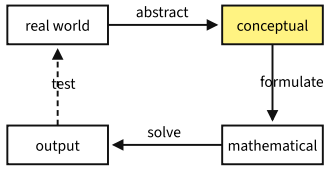
PROCESS	DESIGN RESPONSE
Dilution	measure discharge
pH / adsorption	analyse dissolved + particulate
Settling / storage	allow for later resuspension
Tide	complete spring + neap cycles
Storm	acknowledge rare unsampled export

Consent: report the result as a confidence interval; if it crosses the limit, the centre alone cannot settle compliance. Negative net flux during one spring tide can indicate temporary landward storage, not the long-run state.

SENSE CHECK *Exact seconds/day leave fractional error unchanged.*

6 · Modelling Process

CH 6



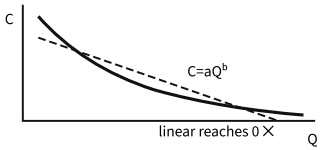
Numerical solution → verify; tune plausible parameters → calibrate; compare with observations → validate.

STAGE	QUESTION
Abstraction / assumption	What is omitted or simplified?
Formulation	How is the concept written mathematically?
Solution	Exact analytical or stepped numerical?
Verification	Does smaller Δt/Δx materially change it?
Calibration	Which plausible parameter values fit?
Validation	Does output match independent reality?

Verification ≠ validation. Verification checks numerical approximation; validation tests assumptions against observations. Analytical solutions need no numerical verification. Add complexity only when failed validation identifies a missing process.

7 · Regression & Transformations

CH 7



Dilution is a power law: a straight line falsely reaches zero concentration.

SIGNIFICANCE FROM R^2

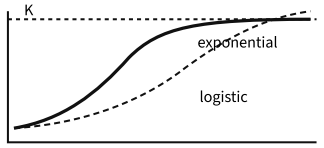
```
r = √r² · df = n-2
t = r√(n-2)/√(1-r²)
p = TDIST(t, n-2, 2); p<0.05 →
significant at 95%
```

Report r^2 with p. Fit strength and significance answer different questions. Power law $y=ax^b$ → double-log straight line; exponential → semi-log. Splitting rising/falling hydrograph limbs captures hysteresis but reduces n, so recompute p.

RESULT	READ IT AS
large r^2, p≥.05	fit without enough evidence
small r^2, p<.05	real but weak relation

8 · Growth Models

CH 8



Logistic growth follows exponential early, then the moderating term forces N toward K.

EXPONENTIAL

```
dN/dt = rN · Nt = N0e^rt · lnNt = lnN0 +
rt
```

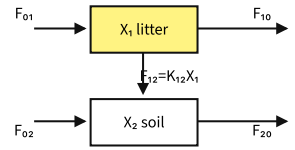
LOGISTIC

```
dN/dt = rN(K-N)/K
Nt = KN0 / [N0+(K-N0)e^-rt]
```

Read N_0 and plateau **K** from data. While N≪K, r≈ $(\ln N_2 - \ln N_1)/(t_2 - t_1)$. Natural logs match e; multiply a base-10 gradient by 2.303. If calibration fails, revisit assumptions.

9 · Compartments

CH 9



Donor-controlled flux: $F_{ij}=K_{ij}X_i$. Write each store as inputs minus outputs.

```
X1' = X1 + Δt[F01-(K10+K12)X1]
X2' = X2 + Δt[F02+K12X1-K20X2]
```

Use old X_1 and old X_2 on both right-hand sides. Equilibrium: set changes to zero; $X_1^*=F_{01}/(K_{10}+K_{12})$, $X_2^*=(F_{02}+K_{12}X_1^*)/K_{20}$. Residence time = 1/ΣKout. Check $K\cdot\Delta t<1$; halve Δt and compare.

OUTPUT CLUE	ACTION
logistic still misses	revisit model structure
store below X* and rising	system is a sink
ΣKout·Δt>1	refine the step
Δt/2 changes result	not verified

10 · Fick Difference Step

CH 10

```
Fi→i+1 = Kd(Ci-Ci+1)
Ci(t+1)=Ci(t)+Kd[Ci-1-2Ci+Ci+1]
```

Check total mass each step. The guide's grid has a limiting Kd about **0.33**; above it the neighbour overtakes the centre, and beyond 0.5 the centre becomes negative. For a conservative spike, variance = 2Kd·t.

Online open-book test (2 × 1.5 hours) after teaching week 12, Semester 1 2026/27 (check the date in your exam timetable).
Check your unit's exam rules for any limit on what may be brought in.

EART60061 Worked Steps · Traps · Fast Recall

MEASURING AND PREDICTING 1 · THE UNIVERSITY OF MANCHESTER

Full guide + ask Sia about EART60061

SIDE 2/2 WORKED EXAMPLES · COMMON ERRORS · METHOD CHOOSER

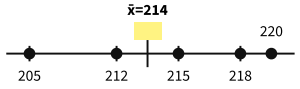
OPEN-BOOK EXAM SHEET

Keep calculations + exercise files organised

11 · Worked: Sample Spread

CH 2 EXAMPLE

Data: conductivity = 212, 220, 205, 218, 215 µS/cm.



δ: −9, −2, +1, +4, +6

All deviations are measured from 214; signed deviations cancel, so use absolute values or squares.

1. $\bar{x}$ = 1070/5 = **214 µS/cm**.
2. Deviations: −2, 6, −9, 4, 1; Σ=0.
3. MAD=(2+6+9+4+1)/5=**4.4 µS/cm**.
4. Squares: 4+36+81+16+1=138.
5. s=√(138/4)=√34.5=**5.87 µS/cm**.

Trap: divisor is n−1=4 for a sample. The largest deviation (−9) receives extra weight after squaring.

12 · Drill: Normal Tail

CH 3 EXAMPLE

River temperature: x̄=14.2°C, s=1.8°C.

```
95%: 14.2 ± 1.96(1.8)=14.2 ± 3.53
10.67 to 17.73°C
```

Threshold 18.0°C: d=(18.0−14.2)/1.8=**2.11**. Upper-tail area ≈0.017 → **1.7%** expected above 18.0°C.

PROMPT	AREA
within ±d	central area
above +d	one upper tail
outside ±d	both tails

SAY IT *This estimate assumes normal random variation and that the sample mean and s describe the population.*

FAST DRILL: SEVEN-FOOT LOGIC

The guide's practice check uses x̄=171 cm, s=6.2 cm and threshold 185 cm. d=(185-171)/6.2=2.26; the upper-tail area is about 0.012, so roughly **1.2%** exceed 185 cm. In Excel: `1-NORMDIST(185,171,6.2,TRUE)`.

SANITY CHECK	EXPECTED
Σ(xᵢ−x̄)	0 (rounding aside)
Σ frequency classes	n
normal-curve total area	1
larger s, same n	wider/lower curve
larger threshold d	smaller tail

Resolution: a coarse rectangle sum can give about 73% instead of 68%; refine class width. Density is not class frequency.

12b · Drill: Aim → Objectives

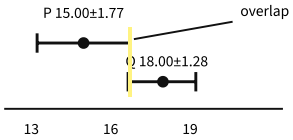
CH 1 EXAMPLE

Bypass → swimming lake. General introduction: chloride harm → de-icing source → drainage path → recreational use.

13 · Worked: Two 95% Means

COURSE EXAMPLE

Seaweed copper (mg/kg): P = 14.2,15.8,13.9,16.1; Q = 17.6,18.9,17.1,18.4. n=4, df=3, t=3.182.



The intervals overlap only from 16.72 to 16.77—but overlap is still overlap.

	P	Q
x̄	60/4=15.00	72/4=18.00
s	1.111	0.804
SE=s/2	0.555	0.402
CL=t·SE	1.77	1.28
interval	13.23–16.77	16.72–19.28

Decision: intervals overlap slightly; a difference cannot be claimed at 95% confidence. Do not judge by means alone.

13b · Confidence Traps

CH 4

- Do not use SD as the error of a mean.
- Do not use 1.96 when small n requires t.
- df for a mean is n−1; regression uses n−2.
- TINV first argument is outside probability 0.05.
- Mean and limit: same units and decimal places.
- A bar without stated confidence is uninterpretable.

Precision grows as √n: quadrupling n halves SE, all else equal.

THRESHOLD CHECK: LEAD GUIDELINE

Six soil replicates: x̄=31.4 mg/kg, s=2.6 mg/kg; t=2.571 for df=5. SE=2.6/√6=1.061; CL=2.571×1.061=**2.73 mg/kg**. Interval 28.67–34.13 lies wholly below 35, so the mean is below the guideline at 95% confidence.

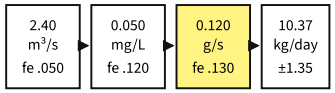
GRAPH CHOICE	REASON
scatter by distance	shows spatial relation required by objective
95% error bars	make uncertainty/comparison visible
reference/consent line	turns plot into a decision
points unconnected	no claim between sampled sites
legend below	states bars and units unambiguously

Opposing corrections: replacing s with SE shortens the bar because means vary less than readings; replacing 1.96 with small-sample t lengthens it because few replicates give less confidence. The correct bar needs both.

14 · Worked: Nickel Load

CH 5 EXAMPLE

Given: Q=2.40±0.12 m³/s; C=0.050±0.006 mg/L.



concentration error dominates

Separate the number-and-unit chain from the fractional-error chain.

1. Flux=2.40×0.050=**0.120 g/s**.
2. Fractional errors: 0.12/2.40=0.050; 0.006/0.050=0.120.
3. Combined fe=√(0.050²+0.120²)=√0.0169=**0.130**.
4. Daily=0.120×86,400=10,368 g=**10.37 kg/day**.
5. E=0.130×10.37=**1.35 kg/day**.

Answer: 10.37±1.35 kg/day (equivalently 0.120±0.016 g/s). Improve concentration measurement first because its fractional error is larger.

14b · Rule-and-Unit Traps

CH 5

MISTAKE	CORRECTION
sum errors for A plus B	quadrature of absolute E
product errors in units	quadrature of fractional E
give 86,400 an error	exact; fe unchanged
judge consent by centre	compare full interval
negative tidal flux = wrong	can mean net landward flux

Sampling logic: cover predictable tidal cycles at spring and neap; acknowledge unsampled storm variation and storage/resuspension.

WRITE EVERY PROPAGATION ANSWER IN TWO PASSES

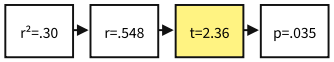
1. List each measured quantity as value, units, absolute error and fractional error.
2. Calculate the central result with units; do not mix error arithmetic into this line.
3. Choose absolute quadrature for addition/subtraction or fractional quadrature for multiplication/division.
4. Convert the final fractional error back to an absolute error in the answer's units.
5. Identify the dominant contribution and compare the whole interval with the criterion.

OPERATION	CARRY	QUICK CHECK
A±B	absolute E	answer E exceeds each component E
A×B or A/B	fractional E	answer fe exceeds each component fe
x^n	|n| times fe	negative n does not make negative error
× exact constant	same fe	absolute E scales with result

15 · Drill: Regression Significance

CH 7 EXAMPLE

Given: n=15 paired samples, r^2=0.30.



df=n−2=13 · two-tailed

r^2 explains variation; p tests evidence. Report both.

1. r=√0.30=**0.548**; df=15−2=**13**.
2. t=0.548√13/√0.70 = 0.548×3.606/0.837 = **2.36**.
3. TDIST(2.36,13,2) gives **p=0.035**.
4. p<0.05: significant at 95%, although discharge explains only 30% of sulphate variation.

High r^2 need not be significant when n is small: the guide's four-point check has r^2=0.85 but p=0.078.

15b · Transform / Split / Recheck

CH 7 CASE

PATTERN / LAW	PLOT OR ACTION
$y=ax^b$	log y vs log x
$y=ae^{bx}$	ln y vs x
time loop	join points in time; split limbs
two solute sources	separate process contributions

A straight concentration–discharge line crossing zero is physically impossible under pure dilution. Use the process to choose the law. Splitting limbs captures depletion/history; each smaller subset has new df and needs its own p.

Case sequence: time series → power-law fit → time-ordered loop → split limbs → remove H^+ contribution → refit and recompute p. Name the physical assumption relaxed at each change.

Different limbs at the same Q indicate solute depletion: ***hysteresis***, not random scatter. A useful rating curve may predict while also revealing sources and storage.

16 · Drill: Logistic Prediction

CH 8 EXAMPLE

Data: N_0=8, N_4=20, plateau K≈400.

```
r=(ln20-ln8)/4=0.916/4=0.229 month⁻¹
N₁₀=400·8/[8+392e^(−2·29)]=3200/47.7≈67
exponential: 8e^(2·29)≈79
```

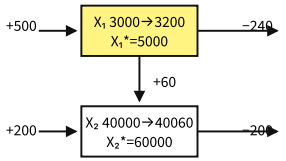
Logistic is lower because resources already moderate growth. Estimate r only from the early straight segment of lnN vs t; then calibrate r and K, and validate against observations.

LIMIT / PLOT	MEANING
N≪K	nearly exponential
N≈K	growth stops
early lnN straight	gradient=r
plateau	estimate K

17 · Worked: Two Compartments

CH 9 EXAMPLE

F_{01}=500, F_{02}=200 units/y; K_{10}=.08, K_{12}=.02, K_{20}=.005 y^{-1}; X_1=3000, X_2=40,000; Δt=1 y.



Year-one fluxes use starting stores: $X_1 \to X_2$ is 0.02×3000=60.

1. X_1'=3000+500−(.08+.02)3000=**3200**.
2. X_2'=40000+200+.02(3000)−.005(40000)=**40060**.
3. X_1*=500/.10=**5000**.
4. X_2*=[200+.02(5000)]/.005=**60,000**.

Both stores are below equilibrium and increasing: soil is a **sink**. Residence times: X_1=1/.10=10 y; X_2=1/.005=200 y. Never update X_1 then reuse the new value in X_2 during the same explicit step.

17b · Verify Before Trusting

CH 9

CHECK	EVIDENCE
coefficient	every ΣKout·Δt<1
time step	halve Δt; difference immaterial
boundary	long-run output = analytical equilibrium
balance	inputs−outputs explain Δstore

The course demonstration found one-year vs half-year steps differed by under 1%; smaller steps brought no useful benefit relative to input accuracy. State both the difference and why it is acceptable.

18 · Model-Term Decoder

CH 6

One mixed reach = **abstraction**. Heat gain ∝ sunshine = **assumption**. Heat-balance equation = **formulation**. Hourly spreadsheet = **numerical solution**. Half-hour rerun = **verification**. Tune shading factor = **calibration**. New-period comparison = **validation**.

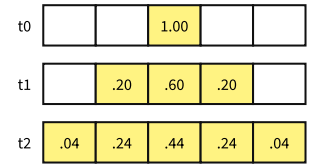
TERM PAIR	DO NOT CONFUSE
conceptual / mathematical	process description / equations
analytical / numerical	exact integrated form / stepped approximation
verification / validation	solve equations right / equations represent reality
calibration / validation	tune plausible values / test resulting model
deterministic / probabilistic	one fixed answer / distribution from parameter ranges

Start simple. Failure may expose sensitivity or a missing process; an engineering model must predict for its purpose.

19 · Worked: Diffusion Two Steps

CH 10 EXAMPLE

Unit mass starts in the centre; Kd=0.2.



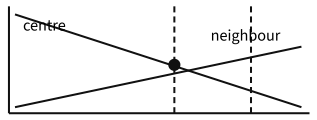
Explicit diffusion spreads symmetrically while the cell total stays exactly one.

1. t1: centre=1−2(.2)=**.60**; neighbours=.20.
2. t2 centre=.6+.2(.2−1.2+.2)=**.44**.
3. t2 neighbours=.2+.2(.6−.4+0)=**.24**; outer=.2(.2)=**.04**.
4. Mass=.44+2(.24)+2(.04)=**1.00**.
5. Variance=2(.24)(1²)+2(.04)(2²)=**.80=2Kd·t**.

The centre remains highest because Kd<⅓. A drifting total means a flux was omitted or counted wrongly. A degrading pollutant would deliberately lose mass; a conservative one must not.

19b · Stability Picture

CH 10 FIGURE



At Kd≈0.33 neighbour overtakes centre; beyond 0.5 the centre becomes negative.

$F_{i\to i+1}=Kd(C_i-C_{i+1})$. Update every cell from the same old row. Negative or alternating concentrations are numerical artefacts.

Random walk: 500 particles move one unit left/right per step; the frequency becomes normal and variance grows with iteration. Particle count must stay fixed.

CHECK	PASS
symmetry	equal neighbours
mass	ΣC constant
spread	variance=2Kd·t
stability	Kd below limit

20 · Final Method Chooser

ALL CHAPTERS

CLUE	WRITE FIRST
replicate spread	s with n−1
rare threshold	d=(x−x̄)/s; tail
uncertain mean	t·s/√n
A±E plus B±E	absolute quadrature
A±E times B±E	fractional quadrature
line significance	r^2→r→t, df n−2→p