PHYS2250 Introductory Mechanics

THE UNIVERSITY OF HONG KONG · LEVEL 2 · FIRST SEMESTER 2026–27

Full guide + ask Sia about PHYS2250

SIDE 1/2 METHODS · assumptions → law → constraint → check

ONE A4 NOTE SHEET

Final: two-hour written examination · 50%

01 · Exam map COURSE DATA

Final written examination 50% | Mid-term 25% | Labs 15% | Assign

"Students may bring a single A4-sized paper to the midterm and final exams with printed or written notes on both sides." *PHYS2250 Moodle "Cheat Sheet" page, first semester 2026-27.*

PROMPT SIGNAL	FIRST MOVE	CLOSE WITH
trajectory / time	components + shared t	boundary
forces / contact	one-body FBD	N,T ≥ 0 phase
short impact	system + impulse	collision condition
speed from endpoints	energy account	transfer + units
rotation / impact	name axis/origin	torque/impulse audit

represent → choose law → solve/check

constraint / boundary links equations

Long-problem route: the constraint is usually the decisive link.

EXAM *Keep symbols until the final substitution; check dimensions, signs, limits and physical direction.*

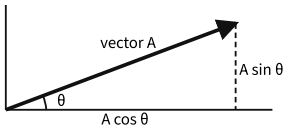
02 · Vectors + dimensions C1

AUDIT BEFORE ALGEBRA

$[v]=LT^{-1}$ · $[a]=LT^{-2}$ · $[p]=MLT^{-1}$ · $[E]=ML^2T^{-2}$

$A_x=A\cos\theta$ · $A_y=A\sin\theta$ · $|A|=\sqrt{(A_x^2+A_y^2)}$

Add only like dimensions; trig/exponential arguments are dimensionless. Agreement is necessary, not proof. A negative component means opposite the chosen positive axis, not a negative magnitude.



Resolve only after choosing axes that simplify the physical constraint.

Energy + gravitation C9–10

$W=\int F\cdot dr=\Delta K$ · $F=-dU/dx$ · $P=F\cdot v$

K+U conserved only when no non-conservative transfer occurs

SIGNAL	METHOD / TEST
endpoints, unknown path time	energy state account
rough path	friction uses path length; gravity uses height
potential curve	K=E−U; allowed where E≥U
equilibrium	dU/dx=0; minimum stable, maximum unstable

03 · Kinematics C2

DERIVATIVE / INTEGRAL CHAIN

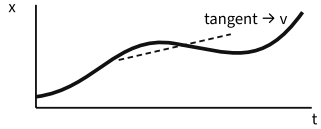
$v=dr/dt$ · $a=dv/dt=d^2r/dt^2$

$\Delta r=\int v\,dt$ · $\Delta v=\int a\,dt$

CONSTANT A ONLY

$v=u+at$ · $s=ut+\frac{1}{2}at^2$ · $v^2=u^2+2as$

On an x–t graph, tangent slope is instantaneous v; changing slope means non-zero a. On a v–t graph, signed area is displacement. At a turning point one component of v can be zero while acceleration remains non-zero.



Read velocity locally from the tangent, not from a chord.

PROJECTILE ROUTE

$x=u_xt+\frac{1}{2}a_xt^2$ · $y=u_yt+\frac{1}{2}a_yt^2$

$u_x=u\cos\theta$ · $u_y=u\sin\theta$ · $a_x=0$ · $a_y=-g$

1. Draw axes and resolve u.
2. Write both component equations with one shared t.
3. Impose the landing/wall/incline boundary last.
4. Reject non-physical roots; substitute back.

SITUATION	CORRECT READING
maximum height	$v_y=0$; $a_y=-g$; usually $v_x\neq0$
inclined landing	trajectory ∩ line equation
variable a	integrate; apply initial conditions
linear-drag limit	expand exponential; recover zero-drag path

TRAP *Constant-acceleration formulae fail when acceleration varies. Return to derivatives and integration.*

Oscillations + waves C11–12

$\ddot{x}=-\omega_0^2x$ · $\omega_0=\sqrt{(k/m)}$

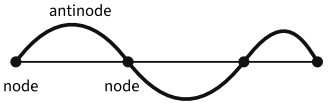
$m\ddot{x}+b\dot{x}+kx=0$

SHM POINT	V	|A|	ENERGY
equilibrium	max	0	K max
endpoints	0	max	U max

Critical damping is the fastest non-oscillatory return. Greater damping lowers and broadens resonance. Natural frequency belongs to the system; the steady response after transients has the driving frequency.

$y=A\sin(kx-\omega t+\varphi)$ · $k=2\pi/\lambda$

$\omega=2\pi f$ · $v=\omega/k=f\lambda$ · $f_{beat}=|f_1-f_2|$

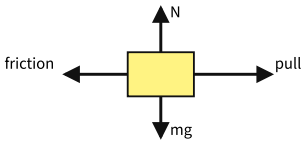


Equal opposite waves form fixed nodes and antinodes.

04 · Newtonian dynamics C3

ONE SELECTED BODY

$\Sigma F=ma$ (vector) → one scalar equation per axis



Every arrow is an external force acting on the isolated block.

1. Select body/system; declare inertial frame.
2. Draw external forces only—no velocity arrow or "force of motion".
3. Choose axes along constraints; project ΣF=ma.
4. Add geometry: string length, contact or pulley relation.
5. Count unknowns/equations; solve; test the assumed phase.

MODEL	WHAT IT EARNS	FAILURE SIGNAL
light inextensible string	fixed length; differentiate twice	slack string
smooth massless pulley	common tension, changed direction	massive/rough pulley
contact surface	N normal to surface	computed N<0
static friction	$\|f\|\leq\mu_sN$, responsive	required $\|f\|>\mu_sN$

CONSTRAINT EXAMPLE

$x+2y=\text{constant} \Rightarrow v_x+2v_y=0 \Rightarrow a_x+2a_y=0$

Third-law partners act on different bodies. They never cancel on a one-body diagram; they cancel internally only when both bodies lie inside the chosen system.

If N=0 or T=0 is reached, end that equation set and re-model the next phase. A surface cannot pull and a slack string cannot impose its former constraint.

EXAM *A scale reads normal reaction, not mass, velocity or direction of travel. Interpret the sign of a independently.*

Idealisation audit C3

GIVEN WORDING	PERMITTED INFERENCE	DO NOT ASSUME
light string	negligible string mass	extensible behaviour
inextensible	fixed total length while taut	equal accelerations without geometry
smooth massless pulley	same ideal tension; direction changes	same rule for massive pulley
ordinary surface	N≥0 while contact persists	surface can pull

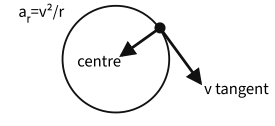
Write the full string length before differentiating; this fixes signs even with moving pulleys. Redraw separate bodies first, then verify internal tensions cancel only in the combined-system balance.

05 · Curved motion + frames C4

NORMAL–TANGENTIAL COMPONENTS

$a_r=v^2/r=\omega^2r$ (towards centre)

$a_t=dv/dt$ · $|a|=\sqrt{(a_r^2+a_t^2)}$



Constant speed removes tangential acceleration, not inward acceleration.

"Centripetal" names the direction of the resultant acceleration; it is not an extra force. Project the real forces inward and set their radial resultant to mv^2/r. Use the tangential balance separately for speed change.

QUESTION	EQUATION / DECISION
level turn	friction may supply mv^2/r; N=mg
impending slip	only then $\|f\|=\mu_sN$
banked turn	resolve N and f; friction direction opposes threatened slip
vertical circle	choose inward positive at each point
straight-path limit	$r\to\infty \Rightarrow a_r\to0$

FRAME DISCIPLINE

FRAME	FORCE LIST
inertial	real forces only; ΣF=ma
rotating	state frame; add appropriate fictitious terms consistently

Never mix the descriptions: adding centrifugal force to an inertial-frame balance double-counts the frame effect. Static friction is determined by the balance first and compared with its limit afterwards.

CHECK *Mass often cancels in a friction-limited level turn because both required and available forces scale with m.*

Turn decision checks C4

AT THE POINT	RADIAL QUESTION	TANGENTIAL QUESTION
constant speed	which real forces provide mv^2/r?	resultant tangential force = 0
changing speed	same inward balance	$\Sigma F_t=m\,dv/dt$
banked surface	resolve N and any f inward	friction opposes threatened slip
rotating observer	state frame before forces	include frame terms consistently

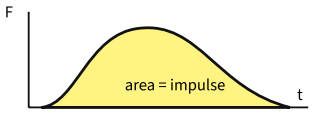
At the top of a vertical circle, choosing inward (downward there) positive gives the shortest equation. Do not preserve a page direction when the radial axis has rotated. On a bank, first solve the frictionless component balance; above or below that design speed, infer the threatened slide before assigning friction direction.

06 · Momentum + collisions C5–6

SYSTEM STATEMENT

$p=mv$ · $J=\int F_{ext}\,dt=\Delta P_{system}$

$\Sigma F_{ext}=dP/dt$



Impulse is signed force–time area, not peak force alone.

1. Declare system and collision interval.
2. List external impulses; justify any neglected during short contact.
3. Conserve total momentum only if net external impulse is zero/negligible.
4. Add exactly the event condition; calculate kinetic energy separately.

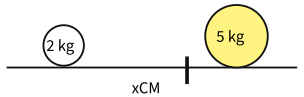
EVENT	EXTRA CONDITION	K AFTER?
stick	common final velocity	decreases
elastic	$K_f=Kf$	same
restitution e	relative separation / approach along impact	derive
smooth oblique	e along normal; tangential handled separately	derive

Momentum conservation does not imply kinetic-energy conservation. State where a decrease goes: deformation, sound and internal energy. For changing mass, define the boundary and include momentum flux; unqualified F=ma generally omits it.

CENTRE OF MASS

$r_{CM}=\Sigma m_ir_i/\Sigma m_i$ · $r_{CM}=\int r\,dm/\int dm$

$M\,a_{CM}=\Sigma F_{ext}$



Mass-weighted position lies closer to the heavier particle.

Symmetry helps only when shape *and density* share it. Choose dm to match geometry; compute numerator and total mass from the same element. Internal forces can change configuration, not complete-system centre-of-mass motion.

Collision + impulse checks C5

$e=(\text{relative separation speed})/(\text{relative approach speed})$

apply along the impact normal

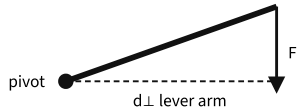
CHECK	MEANING
signed impulse	direction of Δp, not force peak
e condition	adds to momentum; does not replace it
2-D collision	conserve vector components separately
K comparison	classify after momentum solution

07 · Rotation + rolling C7–9

FIXED-AXIS MAP

$v_t=r\omega$ · $a_t=r\alpha$ · $a_r=r\omega^2$

$\tau=r\times F$; $|\tau|=F\,d_\perp$ · $I=\int r^2dm$ · $\Sigma\tau=I\alpha$



Torque uses perpendicular distance to the force line of action.

Name the axis before calculation. Torque and I must refer to the same axis. A force through the chosen origin has zero moment, but it remains in force balance.

QUANTITY	SOURCE-GROUNDED RELATION	CONDITION
axis shift	$I=I_{CM}+Md^2$	parallel axes; one through CM
hinged uniform rod	$I=(1/3)ML^2$	axis at end
solid cylinder	$I=(1/2)MR^2$	central symmetry axis
equilibrium	ΣF=0 and Στ=0	both required

ANGULAR MOMENTUM

$L=r\times p$ (particle) · $L=I\omega$ (fixed principal axis)

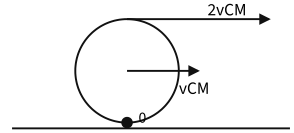
$\int\tau\,dt=\Delta L$ · conserve only if external angular impulse about named origin is zero

Choose an impact origin through unknown impulsive forces so their angular impulse is zero. Conservation may hold about one point and fail about another.

PURE ROLLING

$v_{CM}=R\omega$ · $a_{CM}=R\alpha$

$K=\frac{1}{2}Mv_{CM}^2+\frac{1}{2}I_{CM}\omega^2$



Pure rolling: contact point 0, centre vCM, top 2vCM.

Ideal static friction on a fixed surface does no work at the instantaneous resting contact point, though it may be non-zero. More energy in rotation means less translational speed for the same height drop—not lost energy.

Axis + origin checklist C7–8

BEFORE SCALAR FORMULA	WRITE BESIDE IT
τ=Iα	same named axis for τ and I
$I=I_{CM}+Md^2$	parallel axes; distance d
L=Iω	fixed principal axis
conserve L	external angular impulse zero about origin

PHYS2250 Introductory Mechanics · Worked Drills

NUMBERS TO THE ANSWER · TRAPS · FAST RECALL · METHOD CHOOSER

Full guide + ask Sia about PHYS2250

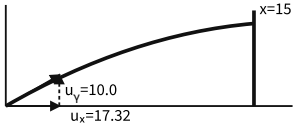
SIDE 2/2 DRILLS · write the law before substituting · preserve units and direction

ONE A4 NOTE SHEET

6 worked calculations · final answers checked

08 · Projectile drill C2 EXAMPLE

Given: u=20 m s⁻¹ at 30°, wall x=15 m, g=9.8 m s⁻². Find y at wall.



Two component equations share one flight time.

1 · RESOLVE
$u_x=20\cos30°=17.32$ m s⁻¹
$u_y=20\sin30°=10.0$ m s⁻¹

2 · HORIZONTAL CLOCK
$15=17.32t \Rightarrow t=0.866$ s

3 · VERTICAL RESULT
$y=10(0.866)-4.9(0.866^2)=$ **4.98 m**

A 4.5 m wall is cleared by 4.98−4.5=**0.48 m**.

Substitution check: the same t must locate both x and y.

TRAP	REPAIR
use total speed in y	use u_y only
add gravity term	$a_y=-g$ for upward-positive
different t in x,y	one event, one clock

09 · Vector micro-drill C1 EXAMPLE

Components 12 N east, 5 N north:

$F=\sqrt{(12^2+5^2)}=$ **13 N**
$\theta=\tan^{-1}(5/12)=$ **22.6° north of east**

Back-check: 13 cos22.6°=12; 13 sin22.6°=5. Never add perpendicular magnitudes to get 17 N.

Method chooser ALL CHAPTERS

IF ASKED FOR…	FASTEST VALID ROUTE	AUDIT
instantaneous force/a	FBD + ΣF=ma	frame, axes, contact
short-impact v	impulse/momentum	system, external impulse
speed from positions	work-energy	all K modes; transfers
rotation after impact	angular impulse	origin, external torque
orbit speed/period	radial balance	centre-to-centre r
wave mode	boundary geometry	nodes, loops, phase

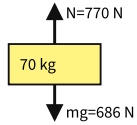
FINISH *State units and physical direction; substitute back into the constraint or boundary.*

Kinematics recall C2

OBSERVATION	TRANSLATION
x–t tangent	instantaneous velocity
v–t signed area	displacement, not total distance
horizontal x–t tangent	v=0 at that instant

10 · Lift + contact drill C3 EXAMPLE

Given: m=70 kg; scale N=770 N; g=9.8 m s⁻². Find acceleration.



The scale reading is N; the force excess determines a.

UPWARD POSITIVE
$N-mg=ma$
$770-70(9.8)=70a$
$84=70a \Rightarrow$ **a=1.20 m s⁻² upward**

The lift may be rising faster *or* descending while slowing. Acceleration does not determine velocity direction.

COMPUTED RESULT	PHYSICAL INTERPRETATION
N>mg	a upward
N=mg	a=0; v may be non-zero
N<mg but N>0	a downward, contact persists
N=0	separation threshold / weightlessness reading
N<0	assumed contact invalid

TRAP *Do not add an upward "motion force"; motion is not an interaction.*

11 · Friction-limited turn C4 EXAMPLE

Level curve r=40 m, μ_s=0.50:

$N=mg$ · $mv^2/r \le \mu_s mg$
$v \le \sqrt{(\mu_s gr)}=\sqrt{[0.50(9.8)(40)]}$
vmax=14.0 m s⁻¹

At 16 m s⁻¹, μrequired=$v^2/(rg)$=0.653>0.50, so no-slip motion is impossible. Mass cancels.

Contact + frame traps C3–4

TEMPTATION	CORRECT TEST
set f=μ_sN immediately	solve required f; compare with μ_sN
add "centripetal force"	sum real inward components
keep contact with N<0	end phase at N=0
mix rotating/inertial views	choose one frame and stay in it
infer v from scale reading	reading determines a only

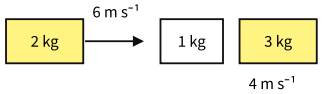
For a vertical circle, redraw inward at top, side and bottom. The radial rule is unchanged even though the positive direction rotates on the page.

Dynamics recall C3

QUANTITY	WHAT IT IS	WHAT IT IS NOT
weight mg	gravitational force	scale reading
normal N	surface contact force	always mg

12 · Inelastic collision C5 EXAMPLE

Given: 2 kg cart at 6 m s⁻¹ sticks to stationary 1 kg cart.



Sticking supplies common final velocity; it does not conserve K.

MOMENTUM
$2(6)+1(0)=(2+1)v$
$12=3v \Rightarrow$ **v=4.0 m s⁻¹**

ENERGY CLASSIFICATION
$K_i=\frac{1}{2}(2)(6^2)=36$ J
$Kf=\frac{1}{2}(3)(4^2)=24$ J
$\Delta K=24-36=$ **-12 J**

Total momentum stays 12 kg m s⁻¹; 12 J becomes deformation, sound and internal energy. A kinetic-energy decrease is not a momentum error.

BEFORE SOLVING	WRITE EXPLICITLY
system	both carts
interval	brief impact
external impulse	negligible horizontally
event condition	lock → common v

13 · Centre-of-mass drill C6 EXAMPLE

2 kg at x=0; 5 kg at x=3.5 m:

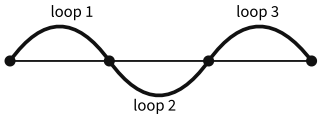
$xCM=[2(0)+5(3.5)]/(2+5)=$ **2.50 m**

First-moment check: 2(2.50)=5.0 kg m and 5(3.50−2.50)=5.0 kg m oppositely. Coordinate average 1.75 m is wrong because masses differ.

Fixed-string drill C12 EXAMPLE

L=1.20 m, wave speed 180 m s⁻¹:

$L=n\lambda/2$ · $\lambda_1=2L=2.40$ m
$f_1=v/\lambda_1=$ **75.0 Hz**
$\lambda_3=2L/3=0.800$ m · $f_3=$ **225 Hz**



Three loops identify n=3; fixed endpoints are nodes.

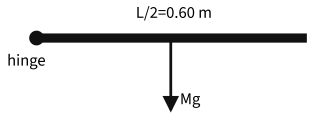
Third harmonic: three loops, two interior nodes. Count loops to identify n before using the boundary formula.

Collision / wave traps C5, C12

CLAIM	VERDICT
momentum conserved in every chosen system	only if net external impulse is zero/negligible
momentum conserved ⇒ K conserved	false; classify K separately

14 · Hinged rod drill C7 EXAMPLE

Given: uniform rod M=3.0 kg, L=1.2 m, horizontal, hinged at one end; released.



Weight acts at the centre; hinge reaction has zero hinge moment.

TORQUE ABOUT HINGE
$\tau=Mg(L/2)=3.0(9.8)(0.60)$
$\tau=$ **17.64 N m**

MATCHING INERTIA
$I_{hinge}=(1/3)ML^2=(1/3)(3.0)(1.2^2)$
$I=$ **1.44 kg m²**

ANGULAR EQUATION
$\tau=I\alpha \Rightarrow \alpha=17.64/1.44$
α=12.25 rad s⁻² in falling direction

TRAP	WHY WRONG
use ICM	torque was taken about hinge
include hinge torque	its lever arm about hinge is zero
Στ=0	rod is released and accelerates
omit direction	sign needs physical interpretation

15 · Inertia / equilibrium recall C7

$I=\int r^2 dm$ · $I=ICM+Md^2$
equilibrium: **ΣF=0 and Στ=0**

Choose a support as torque origin to remove its unknown reaction, solve remaining forces, then return to force components to recover that reaction.

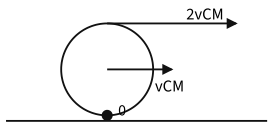
Angular-momentum audit C8

$L=r\times p$ · $L=I\omega$ · $\int \tau dt=\Delta L$

BEFORE CONSERVING L	WRITE
origin	the named point/axis
interval	especially brief impact
external angular impulse	zero/negligible about that origin
unknown hinge impulse	zero moment only if line passes through origin

Conservation can hold about one point and fail about another. A pivot impulse may change linear momentum while giving zero angular impulse about the pivot.

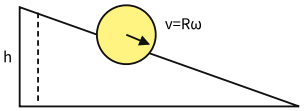
Rotation recall C7–8



Ground-frame speeds: contact 0, centre vCM, top 2vCM.

16 · Rolling + energy drill C8–9 EXAMPLE

Given: solid cylinder rolls from rest through h=1.8 m; $I=(1/2)MR^2$.



Write both kinetic-energy modes before cancelling mass.

ENERGY + NO SLIP
$Mgh=\frac{1}{2}Mv^2+\frac{1}{2}I\omega^2$
$=\frac{1}{2}Mv^2+\frac{1}{2}(\frac{1}{2}MR^2)(v^2/R^2)$
$=\frac{3}{4}Mv^2$

$v=\sqrt{(gh/3)}=\sqrt{[4(9.8)(1.8)/3]}$
v=4.85 m s⁻¹

Sliding particle comparison: $v=\sqrt{(2gh)}$=**5.94 m s⁻¹**. The cylinder is slower because conserved energy is split between translation and rotation.

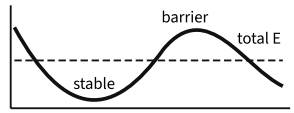
ROUGH-TRACK CONTRAST

4 kg block drops 2.5 m while sliding 6 m against 8 N friction:

$mgh=98$ J · $W_{fric}=-8(6)=-48$ J
$Kf=50$ J $\Rightarrow \frac{1}{2}(4)v^2=50$
v=5.00 m s⁻¹

QUANTITY	USES
gravity	vertical height change 2.5 m
friction work	path length 6 m
turning point	K=0; a may be non-zero
power	P=F·v

17 · Potential curve C9



Allowed where E≥U; intersections are turning points.

$F=-dU/dx$ · $K=E-U$

Minimum: stable equilibrium. Maximum: unstable. Speed is greatest where U is least.

Oscillator drill C11 EXAMPLE

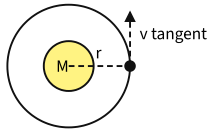
m=0.50 kg, k=72 N m⁻¹, A=0.080 m:

$\omega_0=\sqrt{(k/m)}=$ **12.0 rad s⁻¹**
$T=2\pi/\omega_0=$ **0.524 s**
$v_{max}=\omega_0 A=$ **0.960 m s⁻¹**
$E=\frac{1}{2}kA^2=$ **0.230 J**

At equilibrium speed is greatest and acceleration zero; at endpoints speed is zero and |a| greatest.

18 · Orbit drill C10 EXAMPLE

Given: GM=3.99×10¹⁴ m³ s⁻²; orbital radius r=7.00×10⁶ m.



Gravity is radial; velocity is tangent on a circular orbit.

RADIAL FORCE BALANCE
$GMm/r^2=mv^2/r$
$v=\sqrt{(GM/r)}=$ **7.55×10³ m s⁻¹**

PERIOD
$T=2\pi r/v=2\pi(7.00\times10^6)/(7.55\times10^3)$
T=5.83×10³ s=97.1 min

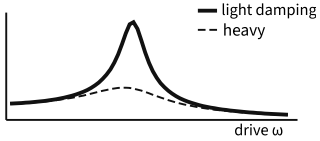
RESULT	FORMULA / MEANING
potential	U=−GMm/r; zero at infinity
circular total E	E=−GMm/(2r)
escape threshold	vesc=$\sqrt{(2GM/r)}=\sqrt{2}$ vcirc
higher circle	lower speed; less-negative total E
uniform shell interior	field zero; potential constant

Radius is from the attracting centre, not altitude above the surface. In a circular orbit gravity does no work because it is perpendicular to instantaneous velocity.

20 · Oscillation / wave recall C11–12

SHM + DAMPING
$x''=-\omega_0^2 x$ · $\omega_0=\sqrt{(k/m)}$
$mx''+bx'+kx=0$

POSITION	SPEED	|ACCELERATION|	ENERGY
equilibrium	maximum	zero	K max
endpoints	zero	maximum	U max



More damping lowers and broadens the resonance peak.

Critical damping is the fastest non-oscillatory return. Driving frequency belongs to the applied force; natural frequency belongs to the unforced system. After transients, steady response has the drive frequency.

TRAVELLING WAVE
$y=A\sin(kx-\omega t+\varphi)$
$k=2\pi/\lambda$ · $\omega=2\pi f$ · $v=\omega/k=f\lambda$

PHASE FORM	DIRECTION
kx−ωt=constant	+x
kx+ωt=constant	−x

Particle velocity $\partial y/\partial t$ is not propagation speed. Add overlapping wave functions before labelling interference. Beats: **$f_{beat}=|f_1-f_2|$**.

Standing waves have fixed nodes and antinodes; adjacent loops oscillate in opposite phase. For a string fixed at both ends, $L=n\lambda/2$. Beats obey $f_{beat}=|f_1-f_2|$.